

$$\text{also } \omega = \frac{2\pi N}{60}$$

$$\left(\frac{2\pi N}{60}\right)^2 = \frac{FIM}{BIM} \cdot \frac{\left[m + \frac{I1}{2}(1+q)\right]}{m} \times \frac{g}{h}$$

$$N^2 = \frac{FIM}{BIM} \cdot \frac{\left[m + \frac{I1}{2}(1+q)\right]}{m} \times \frac{g \times (60)^2}{(2\pi)^2 \times h}$$

$$N^2 = \frac{FIM}{BIM} \cdot \frac{\left[m + \frac{I1}{2}(1+q)\right]}{m} \times \frac{895}{h}$$

Note when $q = R$. Then $q = 1$

$$N^2 = \frac{FIM}{BIM} \cdot \frac{\left[m + \frac{I1}{2}(1+1)\right]}{m} \times \frac{895}{h}$$

$$N^2 = \frac{FIM}{BIM} \cdot \frac{\left[m + I1\right]}{m} \times \frac{895}{h}$$

(1)

Given

$$m = 6.5 \text{ tons} = 6500$$

$$K = 1.8 \text{ m}$$

$$\Delta E = 56 \text{ kN-m} = 56 \times 1000 \text{ N-m}$$

$$N = 120 \text{ r.p.m.}$$

Determined

$$(N_1 \text{ \& } N_2) = ?$$

We know that

$$\Delta E = \frac{H^2}{900} \times m \cdot K^2 \cdot N^2 (s)$$

$$\Delta E = \frac{H^2}{900} \times m \cdot K^2 \cdot N^2 \cdot \frac{(N_1 - N_2)}{N}$$

$$\Delta E = \frac{H^2}{900} \times m \cdot K^2 \cdot N (N_1 - N_2)$$

$$56 \times 10^3 = \frac{H^2}{900} \times 6500 \times (1.8)^2 \cdot (120) (N_1 - N_2)$$

$$N_1 - N_2 = \frac{56 \times 10^3 \times (900)}{H^2 \times 6500 \times (1.8)^2 \times (120)}$$

$$= 2.02065$$

$$N_1 - N_2 = 2 \text{ r.p.m.} \quad \text{--- (1)}$$

We also know that

$$\frac{N_1 + N_2}{2} = N$$

$$N_1 + N_2 = 2N$$

$$= 2(120) = 240 \text{ r.p.m.} \quad \text{--- (2)}$$

$$N_2^2 = \left(\frac{m+11}{m} \right) \frac{895}{h_2}$$

$$= \left(\frac{5+30}{5} \right) \left(\frac{895}{0.15} \right)$$

$$N_2^2 = 41766.6666$$

$$N_2 = 204.3689 \text{ r.p.m.}$$

So,

Range of governor.

$$= N_2 - N_1$$

$$= 204.4 - 177$$

$$= 27.4 \text{ r.p.m.}$$

Also given Friction at the sleeve is equivalent of 20N ($F=20\text{N}$)

We know that

$$N_1^2 = \frac{m \cdot g + (11 \cdot g - F)}{m \cdot g} \times \frac{895}{h_1}$$

$$(N_1)^2 = \frac{5 \times 9.81 + (30 \times 9.81 - 20)}{5 \times 9.81} \times \frac{895}{0.2}$$

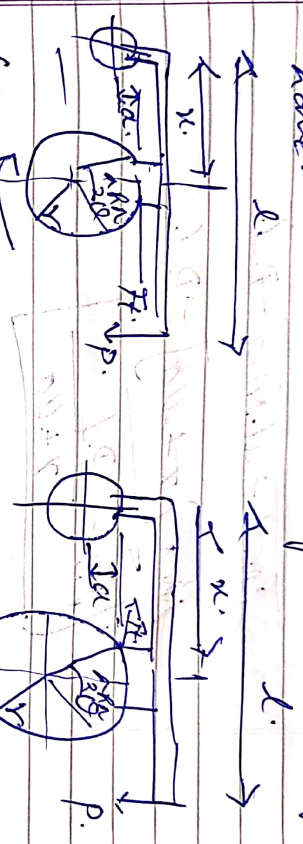
$$(N_1)^2 = 29500.331$$

$$N_1 = 171.7566 \text{ r.p.m.}$$

$$= 172 \text{ r.p.m.}$$

$$(N_2)^2 = \frac{m \cdot g + (11 \cdot g + F)}{m \cdot g} \times \frac{895}{h_2}$$

Case IIIrd When the line of action of the horizontal breaking force (H) passes through a distance a from the fulcrum O , and the bracketed rotates clockwise as shown in figure (a). Then for equilibrium, taking moments about the fulcrum O , we have.



(Clockwise)

(a)

(Anticlockwise)
(b)

$$R_O \times x = P \cdot l + H \cdot a$$

$$= P \cdot l + H \cdot R_O \cdot a$$

$$R_O \cdot x - H \cdot R_O \cdot a = P \cdot l$$

$$R_O [x - Ha] = P \cdot l$$

$$R_O = \frac{P \cdot l}{x - Ha}$$

Breaking Torque

$$T_B = H \cdot R_O \cdot x$$

$$T_B = H \cdot \frac{P \cdot l \cdot x}{x - Ha}$$

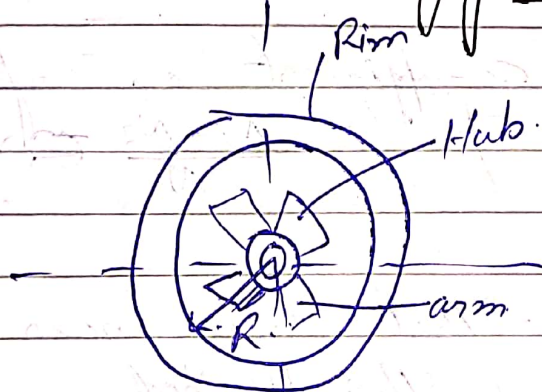
Note

The reciprocal of the Co-efficient of fluctuation of speed is known as Co-efficient of Steadiness and it is denoted by m .

$$m = \frac{1}{C_s}$$

$$= \frac{N}{N_1 - N_2} = \frac{N_1 + N_2}{2(N_1 - N_2)}$$

Energy stored in flywheel.



A flywheel is shown in the figure. We have discussed when flywheel absorb energy its speed increases and when it gives energy its speed decreases.

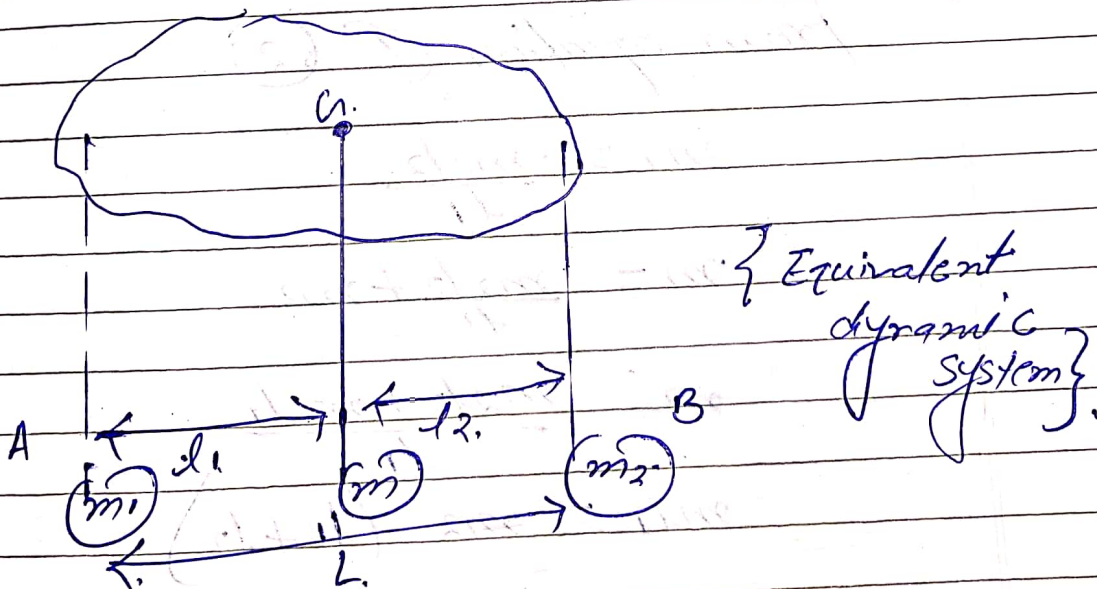
Let m = Mass of the flywheel in kg.
 R = Radius of gyration of flywheel in meters.

I = Mass moment of inertia of the flywheel about its axis of rotation.
 in $\text{kg-m}^2 = mR^2$.

Equivalent dynamic system. [EDS]

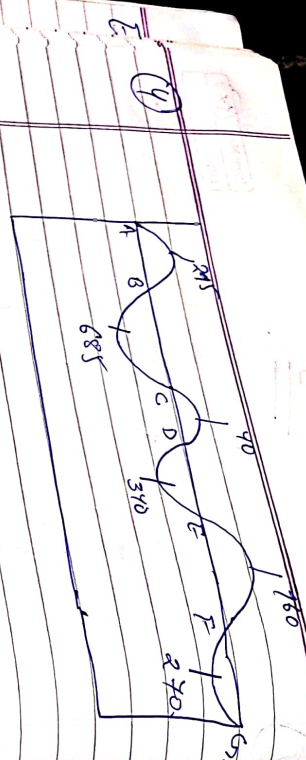
In order to determine the motion of a rigid body under action of external forces, it is usually convenient to replace the rigid body by two masses placed at a fixed distance apart, in such a way that

1. The sum of their masses is equal to the total mass of the body.
2. The centre of gravity of the two masses coincides with that of the body.
3. The sum of masses moment of inertia of the masses about their centre of gravity is equal to mass moment of inertia of the body.



When these three conditions are satisfied then it is said to be an equivalent dynamic system.

A. Centrifugal governors: The centrifugal governors are based on the balancing of centrifugal force on the rotating balls by an equal and opposite radial force, known as the controlling force. It consists of two balls of equal mass which are attached to the arms as shown in figure. These balls are known as governor balls or fly balls. The balls revolve with a spindle which is driven by the engine through bevel gears. The upper end of the arms is pivoted to the spindle, so that the balls may rise up or fall down as they revolve about the vertical axis. The arms are connected by the links to a sleeve which is keyed to the spindle. The sleeve also revolves with a spindle but can slide up and down. The balls rise and fall when the spindle speed increases and falls when the speed decreases.



Given: $m = 36 \text{ kg}$
 $K = 1500 \text{ N/m} = 0.15 \text{ m}$
 $N = 1800 \text{ N/m}$

Let energy at $A = E$.

11 at B = $E + 295 = E - 310$

11 at C = $E + 295 - 685 = E - 350$

11 at D = $E + 295 - 685 + 40 = E - 350$

11 at E = $E + 295 - 685 + 40 + 340 = E - 690$

11 at F = $E + 295 - 685 + 40 + 340 + 270 = E + 270$

11 at G = $E + 295 - 685 + 40 + 340 + 270 = E$

$\Delta E = \text{Maximum energy} - \text{Minimum energy}$

$= E + 295 - E - 690$

$\Delta E = 985 \text{ mm}^2$

But $1 \text{ mm} = 5 \text{ N-m}$

$1 \text{ mm} = 10$

$7124 \times 5 = \frac{54}{180} = \frac{14}{36} \text{ N-m}$

$180 = \frac{54}{36} \text{ N-m}$

$180 = \frac{54}{36} \text{ N-m}$

$180 = \frac{54}{36} \text{ N-m}$

$180 = \frac{54}{36} \text{ N-m}$

$180 = \frac{54}{36} \text{ N-m}$

N_1 and N_2 = Maximum and minimum speeds during the cycle in r.p.m.
 ω_1 and ω_2 = Maximum and minimum angular speed during cycle in rad/sec.

N = Mean speed during the cycle in r.p.m. = $\frac{N_1 + N_2}{2}$

ω = Mean angular speed in rad/sec. = $\frac{\omega_1 + \omega_2}{2}$

C_s = Co-efficient of fluctuation of speed = $\frac{N_1 - N_2}{N}$ or $\frac{\omega_1 - \omega_2}{\omega}$

We know that the mean kinetic energy of the flywheel

$$E = \frac{1}{2} I \omega^2$$

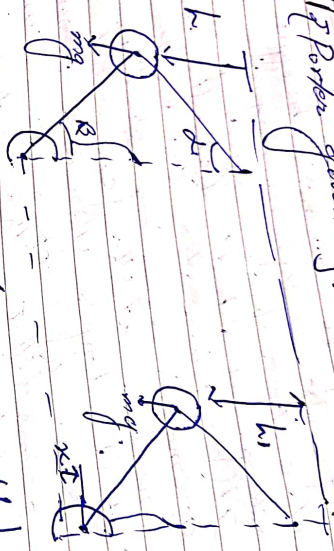
$$= \frac{1}{2} \times m k^2 \times \omega^2$$

As the speed of the flywheel changes from ω_1 to ω_2 the maximum fluctuation energy

$$\Delta E = \text{Maximum K.E.} - \text{Minimum K.E.}$$

$$= \frac{1}{2} \times I (\omega_1)^2 - \frac{1}{2} \times I (\omega_2)^2$$

Q. Effort and Power of a Porter Governor.



(a) Position at equilibrium (b) Position at increased speed

Let N = Equilibrium speed. corresponding to the configuration as shown in fig (a)

C = Percentage increased in speed.

Increase in speed = $C \cdot N$

Total speed = $N + C \cdot N$
 $= N(1 + C)$

Assume the angles α and β are equal ($\alpha = \beta$), then the height of a governor

$h = \frac{m + M}{m} \times \frac{g}{N^2}$ — (1)

If the speed increase to $(1 + C)N$ rpm.

At $\frac{m + M}{m} \times \frac{g}{(1 + C)^2 N^2}$ — (2)

$$\text{But } F_Q = \frac{F_P}{\sin \cos \phi}$$

$$\text{So } F_N = \frac{F_P}{\cos \phi} \cdot \sin \phi$$

$$\boxed{F_N = F_P \cdot \tan \phi}$$

(4) Crank Pin effort and Thrust on crank shaft bearings.

The force acting on the connecting rod, F_Q may be resolved into two components: one perpendicular to the crank and the other along the crank. The component F_Q perpendicular to the crank is known as crank pin effort and it is denoted by (F_T) .

Resolving F_Q Perpendicular to the crank.

$$F_T = F_Q \cdot \sin(\theta + \phi)$$

$$\cos [90 - (\theta + \phi)] = \frac{F_T}{F_Q}$$

$$\sin(\theta + \phi) = \frac{F_T}{F_Q}$$

$$F_T = F_Q \cdot \sin(\theta + \phi)$$

$$F_T = \frac{F_P}{\cos \phi} \sin(\theta + \phi)$$

$$(M_1)^2 = m \cdot g + \left(\frac{m \cdot g + F}{2} \right) (1 + G) \times 875$$

$$= 219.81 + \left(\frac{157.81 + 24}{2} \right) (1 + 0.7536) \times 875$$

$$219.81$$

$$(M_1)^2 = 33635.33 \quad \text{r.p.m.}$$

$$= 183.39 \text{ r.p.m.}$$

$$(M_2)^2 = m \cdot g + \left(\frac{m \cdot g + F}{2} \right) (1 + G) \times 875$$

$$m \cdot g$$

$$(M_2)^2 = 219.81 + \left(\frac{157.81 + 24}{2} \right) (1 + 0.7127) \times 875$$

$$219.81$$

$$(0.153)$$

$$(M_2)^2 = 47630.42 \quad 49482.91 \quad \text{r.p.m.}$$

$$M_2 = 222.4475 \quad \text{r.p.m.}$$

$$\text{Range} = M_2 - M_1$$

$$= 222.44 - 183.39$$

$$\text{Range} = 39.0575 \quad \text{r.p.m.}$$

Q3]

Given:

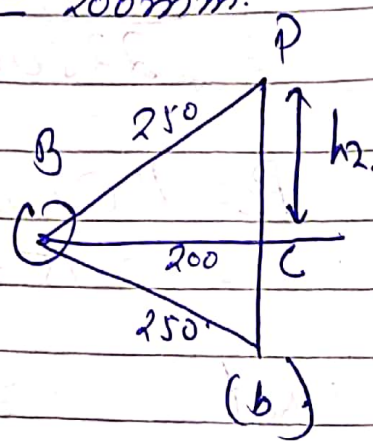
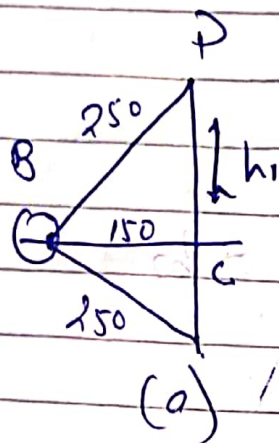
Equal arms = 250 mm.

$m = 5 \text{ kg}$

$M = 30 \text{ kg}$

$r_1 = 150 \text{ mm}$

$r_2 = 200 \text{ mm}$



From (a) value of h_1

From (b) value of h_2

$$PB^2 = BC^2 + PC^2$$

$$PC^2 = PB^2 - BC^2$$

$$PC = \sqrt{PB^2 - BC^2}$$

$$= \sqrt{(250)^2 - (150)^2}$$

$$PC = \sqrt{40000}$$

$$PC = 200 \text{ mm}$$

$$PC = 0.2 \text{ m}$$

$$h_1 = 0.2 \text{ m}$$

$$PB^2 = BC^2 + PC^2$$

$$PC^2 = PB^2 - BC^2$$

$$PC = \sqrt{PB^2 - BC^2}$$

$$= \sqrt{(250)^2 - (200)^2}$$

$$= \sqrt{22500}$$

$$PC = 150 \text{ mm} \times 10^{-3} \text{ m}$$

$$PC = 0.15 \text{ m}$$

$$h_2 = 0.15 \text{ m}$$

We know that

$$(N)^2 = \left(\frac{m + r_1}{m} \right) \frac{895}{h_1}$$

$$N_1^2 = \left(\frac{5 + 30}{5} \right) \frac{895}{0.2}$$

$$N_1^2 = 31325$$

$$N_1 = 176.988 \text{ r.p.m.}$$

$$= \frac{5 \times 9.81 + (30 \times 9.81 + 20)}{5 \times 9.81} \times \frac{875}{0.15}$$

$$(N_2)^2 = 44199.55 \text{ r.p.m.}$$

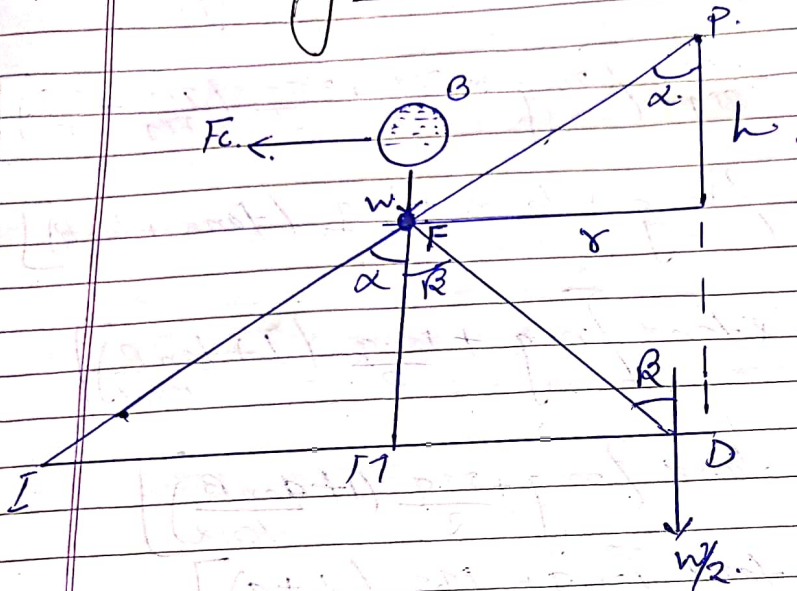
$$N_2 = 210.23 \text{ r.p.m.}$$

Range of governor

$$N_2 - N_1$$

$$= 270 - 172$$

Proell governor



Consider the equilibrium of the forces on one-half of the governor as shown in figure. The instantaneous centre I lies on the intersection of the line PF produced and the line from D drawn perpendicular to the spindle axis. The perpendicular BI is drawn on ID .

Taking moment about I

$$F_c \times BI = W \times IM + \frac{W}{2} \times ID$$

$$= mg \times IM + \frac{Mg}{2} \times ID$$

$$F_c = m \cdot g \times \frac{IM}{BI} + \frac{Mg}{2} \left(\frac{IM \times ID}{BI} \right)$$

2. The work done per cycle is may also be obtained by using the following relation.

$$\text{Work done per cycle} = \frac{P \times V_0}{n}$$

where n = Number of working stroke per minute.

Note: $n = \frac{N}{2}$ in case of two stroke? N in case of four stroke.

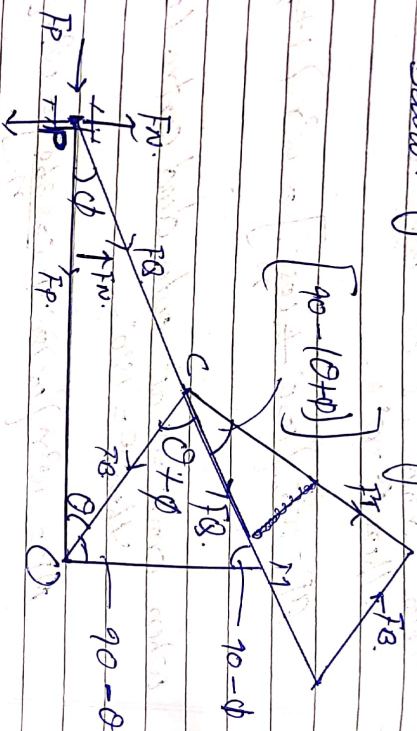
Effect of Flywheel:- A flywheel used in machines stores energy during the period when the supply energy is more than the requirement and release it during the period when the requirement of energy is more than the supply.

In other words, a flywheel control the speed variations depend by the fluctuation of the engine's working moment during each cycle of operation.

Note: The fluctuation of a governor in engine is entirely dependent from that of a flywheel it regulates the mean speed of an engine when there are variations in the load.

11/11

connecting rod / may be derived as below.



WR.

KEK / 1000.

- 1.

It is the red force acting on the piston & or crosshead pin, along the line of stroke. It is denoted by F_P .

Let m -mass of the springing (kg)

$WR = \text{weight of participating neurons} = (n \times R_g)$

Note

Since $\sin^2 \theta$ is very small as compared to n^2 , therefore neglecting $\sin^2 \theta$

So crank effort $T = F_P \times r \left(\sin \theta + \frac{\sin 2\theta}{2n} \right)$
 $= F_P \times OI_1$

where $OI_1 = \frac{\sin \theta + \frac{\sin 2\theta}{2n}}$

$$\Delta E = \frac{985 \times 77}{36}$$

$$= 86.85 \text{ J}$$

$$= 86 \text{ J or N-m}$$

We know that

$$\Delta E = \frac{h^2}{100} \text{ m.k}^2 \cdot N^2 \cdot \text{c.s.}$$

$$C_s = \frac{100 \times \Delta E}{h^2 \cdot \text{m.k}^2 \cdot N^2}$$

$$= \frac{100 \times 86}{(36) \times (0.15)^2 \times (1800)^2}$$

$$C_s = 2.9882 \times 10^{-3}$$

$$= 0.00298$$

$$= 2.9\%$$

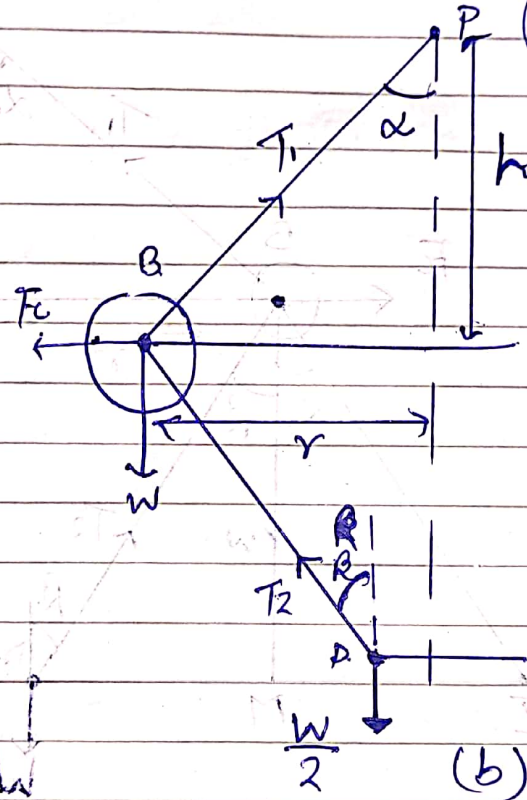
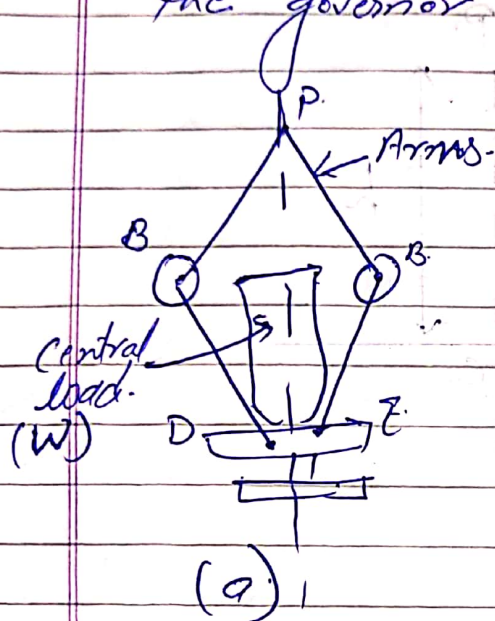
$$= 3\%$$

Q

Porter Governor

The porter governor is a modified of a watt's governor, with central load attached to the sleeve as shown in figure (a).

Consider the force acting on one-half of the governor as shown in figure (b).



Let m = Mass of each ball in kg

w = weight of each ball in newtons = $m \cdot g$

M = Mass of central load in kg

W = weight of central load in newtons = $M \cdot g$

r = Radius of rotation in meters

h = Height of governor

N = Speed of ball in r.p.m

ω = Angular speed of the ball in rad/sec ($\frac{2\pi N}{60}$)

F_c = Centrifugal force

T_1 = Force in the arm newtons

T_2 = Force in the link newtons

When the brake wheel rotates, strain for as shown in fig. moment, about equilibrium, we have

$$R_{ax} + R_{ay} + R_{az} = P \cdot l$$

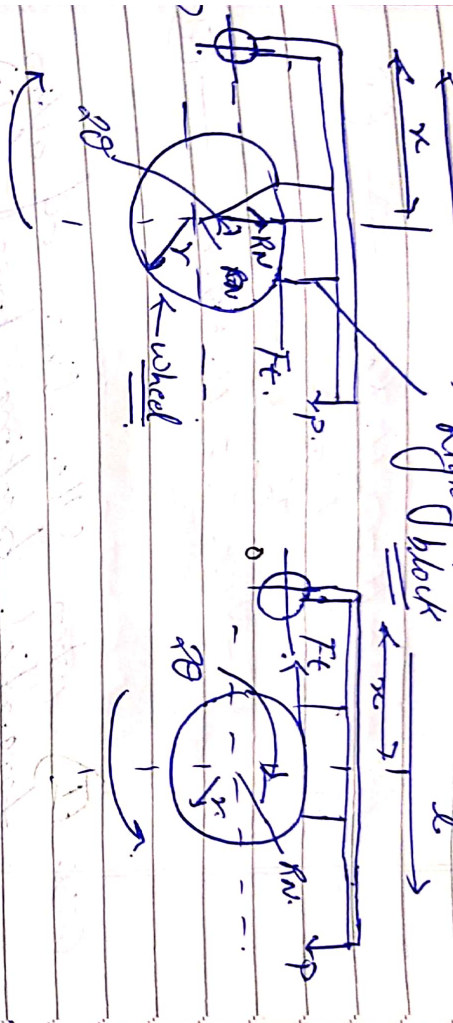
$$R_{ax} [x + a] = P \cdot l$$

$$R_{ax} = \frac{P \cdot l}{x + a}$$

And braking torque,

$$T_B = a \cdot R_{ax}$$

$$T_B = a \cdot \frac{P \cdot l \cdot r}{x + a}$$



* Clockwise rotation of brake wheel
 Anticlockwise rotation of brake wheel.

Let P = Force applied at the end of the lever.

R_n = Normal force pressing the brake block on the wheel.

r = Radius of the wheel.

α = Angle of contact surface of the block.

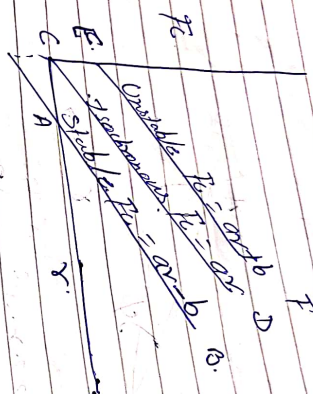
μ = Co-efficient of friction.

F_t = Tangential braking force or the frictional force acting at the contacting surface of the block and the wheel.

θ = Surface of the block and the wheel.

If the angle of contact is less than 60° then it may be assumed that the normal pressure b/w the block and

Controlling Force diagram for spring controlled Governor.



$$F_c = mv^2/r$$

$$F_c = mv^2/r$$

1. For the governor to be stable the controlling force (F_c) must increase as the radius of rotation (r) increases.

2. The relation b/w F_c is for the stability of spring controlled governor is

$$F_c = ax - b \quad \text{--- (1)}$$

3. The value of b in eqn (1) may be made either lower positive by increasing the initial tension of the spring. If b is zero the controlling force line passes through the origin. The governor becomes isochronous and because F_c will remain constant

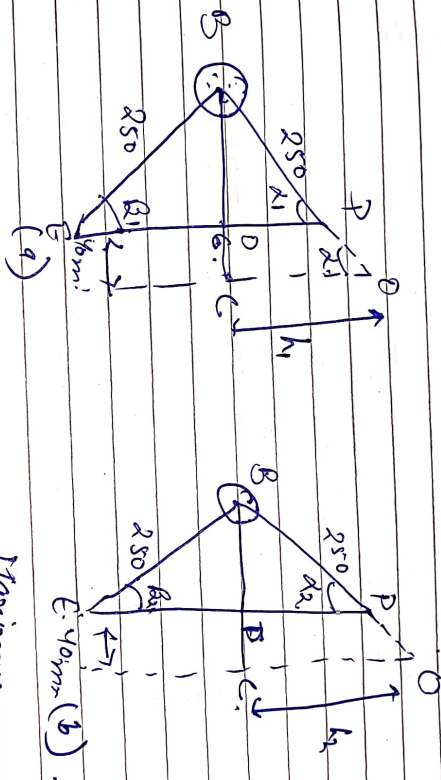
$$N_2 = 164.34 \text{ r.p.m.}$$

$$\text{Range speed} = N_2 - N_1$$

$$= 164.34 - 165.16$$

$$= 4.1812 \text{ r.p.m.}$$

$$= 4.2 \text{ r.p.m.}$$



Minimum.

Maximum.

Given: $m = 3 \text{ kg}$

$$r_1 = 20 \text{ mm}$$

$$r_2 = 150 \text{ mm}$$

$$r_3 = 300 \text{ mm}$$

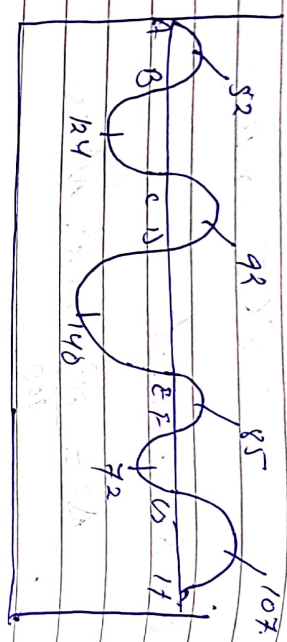
For (a)

$$BD = 150 - 40$$

$$= 110 \text{ mm}$$

(5)

$l = 1 \text{ mm} = 600 \text{ N-m}$
 $1 \text{ mm} = 30^\circ$



$N = 600 \text{ r.p.m.}$
 $r (\text{radius}) = 0.5 \text{ m}$

Let Energy at A = E

Energy at B = E + 52. ✓ Max

" at C = E + 52 - 124 = E - 72

" at D = E + 52 - 124 + 92 = E + 20

" at E = E + 52 - 124 + 92 - 140 = E - 120 Min

" at F = E + 52 - 124 + 92 + 143 - 140 + 85 = E - 35

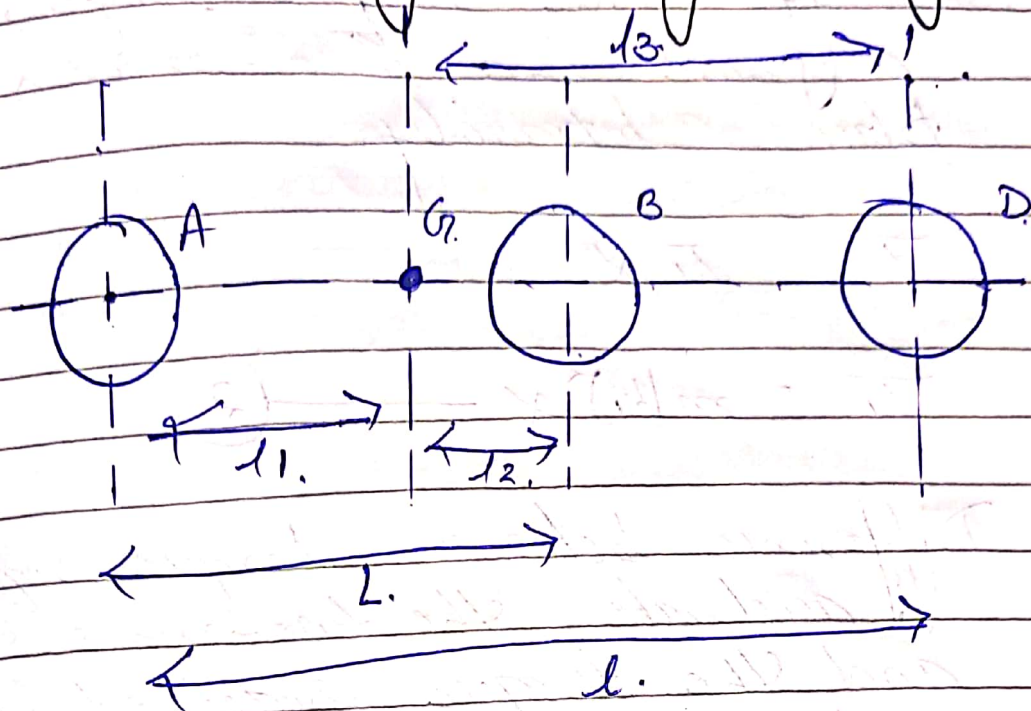
" at G = E + 52 - 124 + 92 + 143 - 140 + 85 - 72 = E - 107

" at H = E + 52 - 124 + 92 + 143 - 140 + 85 - 72 + 107 = E
= Energy at A

$\Delta E = \text{Maximum energy} - \text{Minimum energy}$
 $= E + 52 - E - 120$

$= 52 + 120$
 $= 172 \text{ N-m}$

Correction couple to be applied to make two mass system Dynamically Equivalent



Consider two masses one at A and the other at D to be placed arbitrarily, as shown in figure.

Let l_3 = Distance of mass placed at D from G.
 I_1 = New mass moment of inertia of the two masses

k_1 = New radius of gyration.

α = Angular acceleration of the body.

I = Mass moment of inertia of dynamically equivalent system.

k_G = Radius of gyration of E.D.S.

We know that the torque required to accelerate body

$$T = I\alpha$$

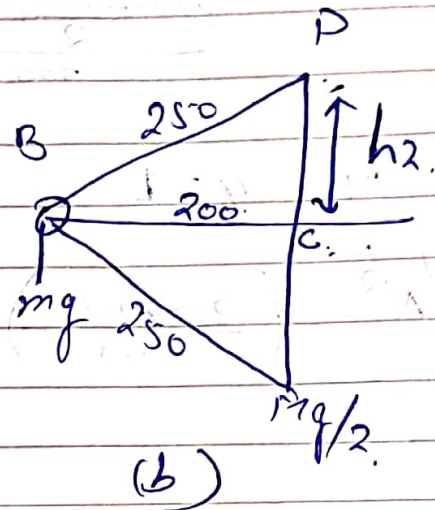
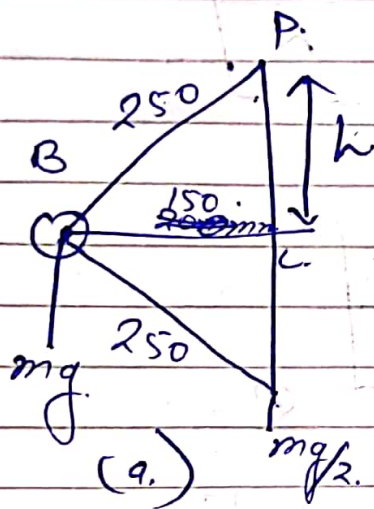
equal arms = 250 mm.

$$m = 5 \text{ kg}$$

$$M = 150 \text{ kg}$$

$$r = 150 \text{ mm}$$

$r = 200 \text{ mm}$ (at maximum speed).



To find L using Pythagoras Theorem in $\triangle PBC$

$$\sin \alpha_1 = \frac{110}{250}$$

$$\sin \alpha_1 = 0.44$$

$$\alpha_1 = 26.1^\circ$$

$$A_1 =$$



while the governor is in equilibrium. For a stable governor if the equilibrium speed increases the radius of governor ball must also increase.

Note. A governor is said to be unstable, if the radius of rotation decreases as the speed increases.

Q Isochronous Governor (Isochronism).

A range of speed should be zero. A governor is said to be isochronous when the equilibrium speed is constant for all radii of rotation of the balls within the working range, neglecting friction.

Let us consider the case of Porter Governor running at speeds N_1 and N_2 (r.p.m.)

$$(N_1)^2 = \frac{m + \frac{M}{2}(1+q)}{\frac{m}{h_1}} \frac{875}{h_1} \quad \text{--- (1)}$$

$$(N_2)^2 = \frac{m + \frac{M}{2}(1+q)}{\frac{m}{h_2}} \frac{875}{h_2} \quad \text{--- (2)}$$

For isochronism range speed should be zero. i.e. $N_2 - N_1 = 0$

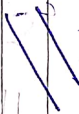
$N_2 = N_1$ i.e. $h_1 = h_2$ which is impossible. Hence a Porter governor cannot be isochronous.

For isochronous.

$$T_c = 2\pi r$$

3. If b greater than zero or positive then T_c decreases as r increases so that the equilibrium speed of the governor decreases with an increase of the radius of rotation of balls, which is impracticable, such a governor is said to be unstable.
and motion becomes.

$$T_c = 2\pi r b$$



$$\text{Acceleration} = \frac{dv}{d\theta} \cdot \frac{d\theta}{dt}$$

$$= \omega r \left(\cos\theta + \frac{\cos 2\theta}{n} \right) \cdot \omega$$

$$\boxed{AP = \omega^2 r \left(\cos\theta + \frac{\cos 2\theta}{n} \right)}$$

Note

(1)

when crank is at inner dead centre
then $\theta = 0^\circ$

$$AP = \omega^2 r \left[\cos 0^\circ + \frac{\cos 0^\circ}{n} \right] = \omega^2 r \left[1 + \frac{1}{n} \right]$$

(2) when crank is at outer dead centre (O.D.C.)
then $\theta = 180^\circ$

$$AP = \omega^2 r \left[\cos 180^\circ + \frac{\cos 360^\circ}{n} \right] = \omega^2 r \left[-1 + \frac{1}{n} \right]$$

As direction of motion reverse at the outer dead centre therefore changing the sign of the equation.

$$AP = \omega^2 r \left[1 - \frac{1}{n} \right]$$

* Terms used in Governors.

1. Height of governor: It is vertical distance from the centre of the ball to a point where the arms of the arm intersect on the spindle axis. It is usually denoted by (h) .
2. Equilibrium Speed: It is the speed at which the governors balls, arms etc. are in complete equilibrium and the sleeve ^{does} not tend to move upwards or downwards.
3. Mean equilibrium speed: It is the speed at the mean position of the balls or the sleeve.
4. Maximum and Minimum equilibrium speed: The speed at the maximum and minimum radius of rotation of the balls, without tending to move either way are known as maximum and minimum equilibrium speeds respectively.
5. Sleeve lift: It is the vertical distance which the sleeve travels due to change in equilibrium speed.

Equating (1) & (2)

$$m + n = \frac{m + n}{(1 + c)^2}$$

$$m = (m + n) \frac{(1 + c)^2}{2}$$

$$m = \frac{m + n}{2} \times (1 + c)^2$$

$$m - n = (m + n) \frac{(1 + c)^2}{2} - m - n$$

$$m - n = (m + n) \frac{(1 + c)^2}{2} - (m + n)$$

$$m - n = (m + n) \left[\frac{(1 + c)^2}{2} - 1 \right]$$

(3)

Let E = effect of governor.

$$E = \frac{(m - n)}{2} \cdot g$$

$$= \frac{(m + n) \left[\frac{(1 + c)^2}{2} - 1 \right] \cdot g}{2} \quad \text{from (3)}$$

$$= \frac{(m + n) [1 + c^2 + 2c - 2] \cdot g}{2}$$

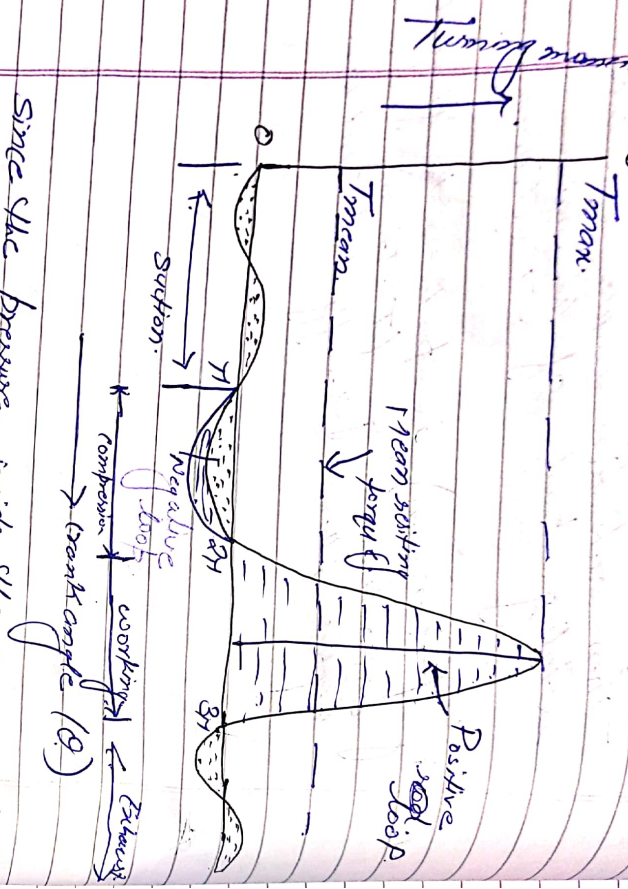
[Neglecting c^2 bcz very small]

$$E = \frac{(m + n) \cdot 2c \cdot g}{2} \quad | \quad E = c \cdot (m + n) \cdot g$$

$$= (m + n) \cdot c \cdot g \quad | \quad = c (mg + nq + F)$$

when friction force is supplied.

Q. Turning moment diagram for a 2-stroke cycle internal combustion engine.



Since the pressure inside the engine cylinder is less than the atmospheric pressure during the suction stroke, therefore a negative loop is formed as in figure. During the compression stroke, the work done on the gases, therefore a higher negative loop is obtained. During the expansion or working stroke the fuel burns and the gas expands, therefore a large positive loop is obtained. In this stroke the work is done by the gases. During exhaust stroke, the work is done on the gases, therefore a negative loop is formed. It may be noted that the effect of the inertia forces

$$R_N x_c + F_N x_a = P \cdot l$$

$$R_N x_c + u \cdot R_N \cdot x_a = P \cdot l$$

$$R_N [x_c + u x_a] = P \cdot l$$

$$R_N = \frac{P \cdot l}{x_c + u x_a}$$

And braking torque

$$= u \cdot R_N \cdot x = \frac{u \cdot P \cdot l \cdot x}{x_c + u x_a}$$

when the brake wheel rotates anticlockwise as shown in figure (b). Then for equilibrium,

$$R_N \cdot x = P \cdot l + u \cdot F \cdot a$$

$$R_N \cdot x = u \cdot R_N \cdot a + P \cdot l$$

$$R_N [x - u a] = P \cdot l$$

$$R_N = \frac{P \cdot l}{x - u a}$$

$$H = \mu \cdot Rv \quad \text{--- (1)}$$

And breaking torque

$$T_B = \tau \cdot r$$

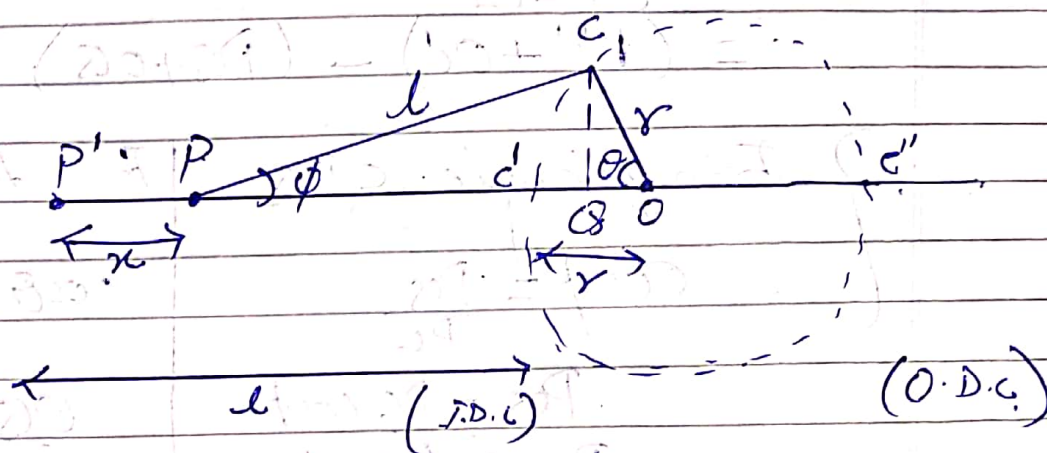
$$= \mu \cdot Rv \cdot r$$

$$\text{--- (2)}$$

(I) when the line of action of tangential braking force (F_t) passes through the fulcrum O of the lever and the brake wheel rotates clockwise as shown in the figure (a) then for equilibrium, taking moments about the fulcrum O we have.

A. Inertia forces in Reciprocating parts.

Extra * Approximate Analytical Method for velocity and acceleration of the piston.



Consider the motion of a crank and connecting rod of reciprocating Steam engine as shown in figure. Let OC be the crank and PC the connecting rod. Let the crank rotates with angular velocity of ω rad/s and the crank turns through an angle θ from the inner dead centre (I.D.C.). Let x be the displacement of reciprocating body P from I.D.C. after time t seconds during which the crank has turned through an angle θ .

Let $l =$ Length of connecting rod b/w centres
 $r =$ Radius of crank or crank pin circle.

Q Sensitivity of Governor:-
The sensitivity is defined as the ratio of difference between the maximum and minimum equilibrium speed to the mean equilibrium speed.

Let n_1 = minimum equilibrium speed
 n_2 = maximum

N = Mean equilibrium speed
 $(\frac{n_1 + n_2}{2})$

Sensitivity of the governor
 $S = \frac{n_2 - n_1}{N}$

$$= \frac{n_2 - n_1}{\frac{(n_1 + n_2)}{2}} = \frac{2(n_2 - n_1)}{n_1 + n_2}$$

In terms of angular speed (ω)

$$= \frac{2(\omega_2 - \omega_1)}{(\omega_1 + \omega_2)}$$

Q Stability of Governor:-

A governor is said to be stable when for every speed within the working range there is a definite configuration i.e. that is there is only one radius of rotation of the governor balls at a

Note:-

SP

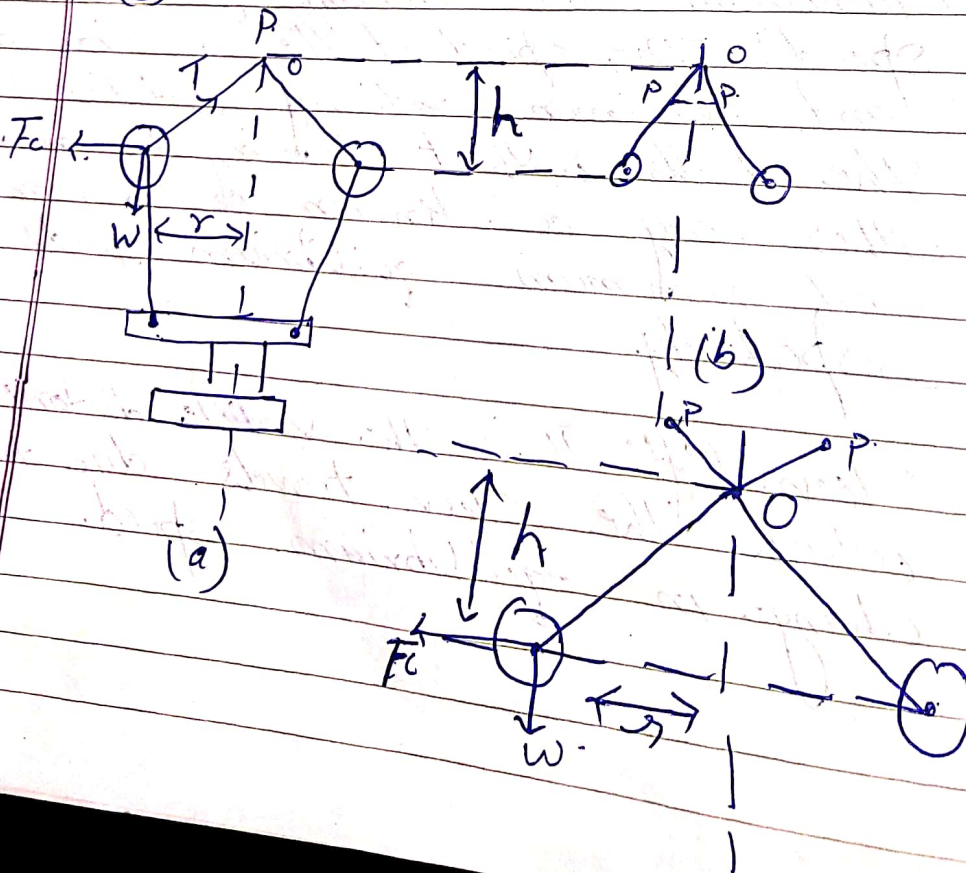
A

SP

stable

Watt Governors.
The simplest form of a centrifugal governor is watt governor as shown in figure. It is typically a conical pendulum with link attached to a sleeve of negligible mass. The arms of the governor may be connected to the spindle in the following three ways.

1. The pivot P may be on the spindle axis as shown in figure (a).
2. The pivot P may be offset from the spindle axis and the arms when produced intersect at O , as shown in figure (b).
3. The pivot P may be offset, but the arms cross the axis of O , as shown in figure (c).



Let:

 $T =$ $W =$ $r =$ $F_c =$ $h =$ $l =$ $l =$ $l =$ $T =$

Multiplying and dividing by F_{11}

$$F_c = \frac{F_{11}}{B_{11}} \left[m \cdot g \times \frac{F_{11}}{F_{11}} + \frac{F_{11} \cdot g}{2} \left(\frac{F_{11}}{F_{11}} + \frac{F_{11}}{F_{11}} \right) \right]$$

$$= \frac{F_{11}}{B_{11}} \left[m \cdot g \times \tan \alpha + \frac{F_{11} \cdot g}{2} (\tan \alpha + \tan \alpha) \right]$$

$$F_c = \frac{F_{11}}{B_{11}} \times \tan \alpha \left[m \cdot g + \frac{F_{11} \cdot g}{2} \left(1 + \frac{\tan \alpha}{\tan \alpha} \right) \right]$$

$$F_c = \frac{F_{11}}{B_{11}} \tan \alpha \left[m \cdot g + \frac{F_{11} \cdot g}{2} (1 + \frac{\tan \alpha}{\tan \alpha}) \right]$$

$$F_c = \frac{F_{11}}{B_{11}} \tan \alpha \left[m \cdot g + \frac{F_{11} \cdot g}{2} (1 + 1) \right]$$

$$\left[\text{where } q = \frac{\tan \alpha}{\tan \alpha} \right]$$

$$\left\{ \begin{array}{l} \text{But } \tan \alpha = \frac{r}{h}, F_c = m r \omega^2 \\ \text{Substitute in (1)} \end{array} \right\}$$

$$F_c = \frac{F_{11}}{B_{11}} \tan \alpha \left[m \cdot g + \frac{F_{11} \cdot g}{2} (1 + 1) \right]$$

$$m \cdot r \omega^2 = \frac{F_{11}}{B_{11}} \left(\frac{r}{h} \right) \left[m \cdot g + \frac{F_{11} \cdot g}{2} (1 + 1) \right]$$

$$\omega^2 = \frac{F_{11}}{B_{11}} \left[\frac{m \cdot g + \frac{F_{11} \cdot g}{2} (1 + 1)}{m} \right] \times \frac{1}{h}$$

$$\omega^2 = \frac{F_{11}}{B_{11}} \left[\frac{m + \frac{F_{11}}{2} (1 + 1)}{m} \right] \times \frac{g}{h}$$

So Equation (7) - (6)

$$(-S_2 - S_1)y = 2 \cdot (F_2 \cdot x) - 2(F_2 - x)$$

$$(S_2 - S_1)y = 2 \cdot x [F_2 - F_1]$$

$$(S_2 - S_1) = 2 \cdot [F_2 - F_1] \frac{x}{y} \quad \text{--- (8)}$$

and $S_2 - S_1 = h \cdot S$, $h = (r_2 - r_1) \frac{y}{x}$

$$S = \frac{S_2 - S_1}{h}$$

So Equation (8) becomes

So $S = 2 \left(\frac{F_2 - F_1}{h} \right) \left(\frac{x}{y} \right)$

$$S = 2 \left(\frac{F_2 - F_1}{r_2 - r_1} \right) \left(\frac{x}{y} \right)^2$$

Note Since for the stiffness for a given spring is constant for all positions, therefore for minimum and intermediate position.

$$S = 2 \left[\frac{F_2 - F_1}{r_2 - r_1} \right] \left(\frac{x}{y} \right)^2 \quad \text{--- (A)}$$

For maximum position and intermediate

$$S = 2 \left(\frac{F_2 - F_1}{r_2 - r_1} \right) \left(\frac{x}{y} \right)^2 \quad \text{--- (B)}$$

on adding (1) & (2)

$$\frac{h_1 + h_2}{y} = \frac{r_2 - r_1}{x}$$

$$h = (r_2 - r_1) \frac{y}{x} \quad \therefore h \neq (h_1 + h_2)$$

(3)

* For minimum Position (Taking moment about O_1)

$$F_{c1} \times x_1 - m \cdot g \times a_1 = \frac{M \cdot g + S_1}{2} \times y_1$$

$$2 [F_{c1} \times x_1 - m \cdot g \cdot a_1] = (M \cdot g + S_1) \times y_1$$

$$\frac{2}{y_1} [F_{c1} \times x_1 - m \cdot g \cdot a_1] = (M \cdot g + S_1)$$

$$S_1 = \frac{2}{y_1} [F_{c1} \times x_1 - m \cdot g \cdot a_1] - M \cdot g$$

(4)

For maximum Position taking moment about O_1

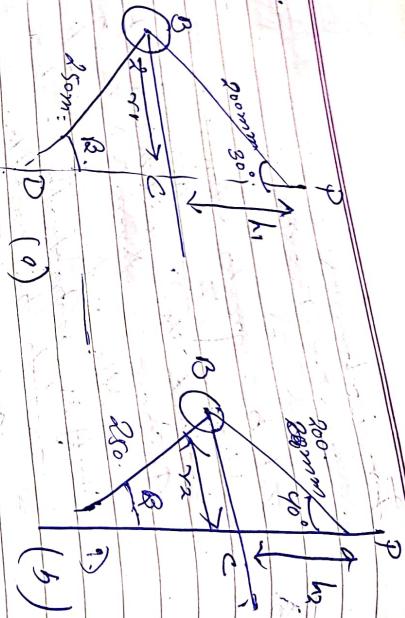
$$F_{c2} \times x_2 + m \cdot g \times a_2 = \frac{(M \cdot g + S_2)}{2} \times y_2$$

$$2 [F_{c2} \times x_2 + m \cdot g \cdot a_2] = (M \cdot g + S_2) \times y_2$$

$$\frac{2}{y_2} [F_{c2} \times x_2 + m \cdot g \cdot a_2] = (M \cdot g + S_2)$$

$$S_2 = \frac{2}{y_2} [F_{c2} \times x_2 + m \cdot g \cdot a_2] - M \cdot g \quad (5)$$

Ex 18.4



Given: $P = 1549$
 $m = 249$
 $F = 249$

$$\sin 30^\circ = \frac{P_1}{200}$$

$$P_1 = \sin 30^\circ \cdot 200$$

$$P_1 = 100 \text{ mm}$$

$$P_1 = 0.1 \text{ m}$$

$$\cos 30^\circ = \frac{h_1}{200}$$

$$h_1 = \cos 30^\circ \cdot 200$$

$$h_1 = 173.20 \text{ mm}$$

$$h_1 = 0.173 \text{ m}$$

$$\sin 40^\circ = \frac{P_2}{200}$$

$$P_2 = \sin 40^\circ \cdot 200$$

$$P_2 = 128.55 \text{ mm}$$

$$\cos 40^\circ = \frac{h_2}{200}$$

$$h_2 = \cos 40^\circ \cdot 200$$

$$h_2 = 153.208 \text{ mm}$$

$$h_2 = 0.1532$$

For

A

CD = 1

CD

CD

f

From Figure (b)

$$P_c = \sqrt{P_B^2 - B_c^2}$$

$$= \sqrt{(250)^2 - (200)^2}$$

$$= \sqrt{22500}$$

$$P_c = 150 \text{ mm}$$

$$= 0.15 \text{ m} = h_2$$

we know that

$$N^2 = \left(\frac{m+1}{m} \right) \cdot \frac{895}{h_1}$$

$$N_1^2 = \left(\frac{5+15}{5} \right) \left(\frac{895}{0.2} \right) = 17900$$

$$N_1 = 133.79 \text{ r.p.m.}$$

$$= 133.8 \text{ r.p.m.}$$

$$N_2^2 = \left(\frac{m+1}{m} \right) \frac{895}{h_2}$$

$$N_2^2 = \left(\frac{5+15}{15} \right) \left(\frac{895}{0.15} \right) = 23866.66$$

$$N_2 = 154.48840 \text{ r.p.m.}$$

$$= 154.5 \text{ r.p.m.}$$

$$\text{Range of speed} = N_2 - N_1$$

$$= 154.4 - 133.8 = 20.7 \text{ r.p.m.}$$

Subtracting (5) - (4)

$$S_2 - S_1 = \frac{2}{y_2} [F_{c2} \cdot x_2 + m \cdot g \cdot a_2] - \cancel{M \cdot g} \\ - \frac{2}{y_1} [F_{c1} \cdot x_1 - m \cdot g \cdot a_1] + \cancel{M \cdot g}$$

$$= \frac{2}{y_2} [F_{c2} \cdot x_2 + m \cdot g \cdot a_2] - \frac{2}{y_1} [F_{c1} \cdot x_1 - m \cdot g \cdot a_1]$$

But we know that

$$S_2 - S_1 = h \cdot S \quad \text{and } h = (x_2 - x_1) \frac{y}{x}$$

$$S = \left(\frac{S_2 - S_1}{h} \right)$$

$$S = \left(\frac{S_2 - S_1}{x_2 - x_1} \right) \cdot \frac{x}{y}$$

Neglecting the obliquity effect (i.e. $x_1 = x_2 = x$ and $y_1 = y_2 = y$) and the moment due to weight of the balls (i.e. $m \cdot g$).

$$\left(\frac{M \cdot g + S_1}{2} \right) x y = F_{c1} \cdot x \quad \text{For minimum}$$

$$(Mg + S_1) x y = (F_{c1} \cdot x) \cdot 2 \quad \text{--- (6)}$$

$$\left(\frac{Mg + S_2}{2} \right) x y = F_{c2} \cdot x \quad \text{For maximum}$$

$$(Mg + S_2) x y = (F_{c2} \cdot x) \cdot 2 \quad \text{--- (7)}$$

 ~~$Mg + S_2$~~

$\omega_1 =$ Angular speed of the governor at minimum radius (rad/s)

$\omega_2 =$ " " " " maximum radius (rad/s) at

$S_1 =$ Spring force exerted on the sleeve at ω_1 (N)

$S_2 =$ " " " " ω_2 (N)

$F_{c1} =$ centrifugal force at ω_1 (N) $= (m r \omega_1^2)$

$F_{c2} =$ " " " " at ω_2 (N) $= (m r \omega_2^2)$

$S =$ stiffness of the spring or the force required to compress the spring by one (mm)

$x =$ length of vertical ball arm of the lever in meters (m)

$y =$ length of horizontal sleeve arm of the lever in (m)

$r =$ Distance of fulcrum O from governor axis on the radius of rotation when the governor is in mid-position (m)

✓ For the minimum position, the compression of the spring or the lift of sleeve h_1 is given by.

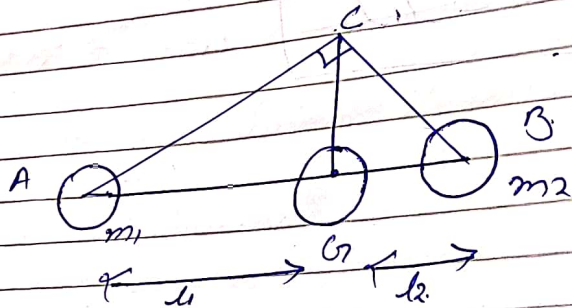
$$\frac{h_1}{y} = \frac{a_1}{x} = \frac{r - r_1}{x} \quad \text{--- (1)}$$

* Similarly for max. position

$$\frac{h_2}{y} = \frac{a_2}{x} = \frac{r_2 - r}{x} \quad \text{--- (2)}$$

Q. Determination of two equivalent masses by Dynamical Graphical method.

Consider a body of mass m acting at G as shown in figure.



This mass m may be replaced by two masses m_1 & m_2 so that the system becomes dynamical equivalent. The position of mass m_1 may be fixed arbitrary at A. Now draw perpendicular CG at G , equal in length of the radius of gyration of the body K_G . Then join AG and draw GB perpendicular to AG intersecting AG produced in B. The point B now fixes the position of the second mass m_2 .

$$(K_G)^2 = l_1 l_2$$

K_G	$=$	$\frac{l_2}{K_G}$
l_1		

Date 14-1-2020

Unit - IIIrd. (1st part)

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Governors.

Introduction: The function of a governor is to regulate the mean speed of a engine, when there are variations in the load e.g. when the load on an engine increases its speed decreases therefore it becomes necessary to increase the supply of working fluid. On the other hand when the load on the engine decreases, its speed increases and thus less working fluid is required.

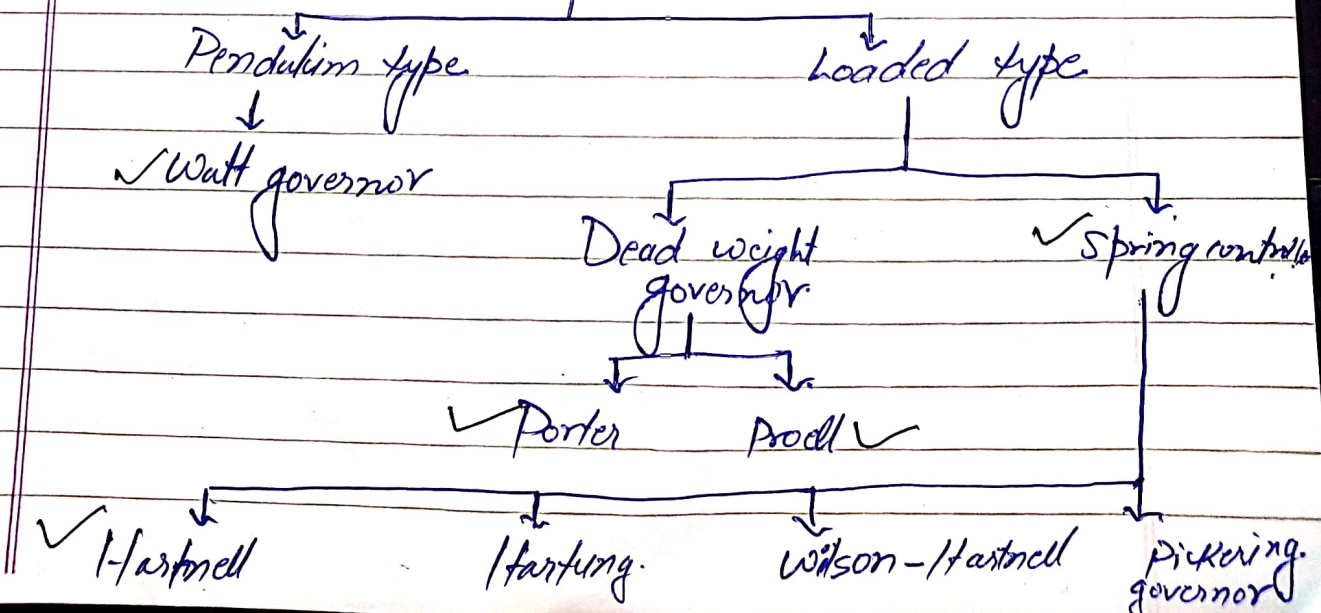
* Types of Governor:

The governor may, broadly, be classified

1. Centrifugal governor.
2. Inertia governor.

The centrifugal governors, may further be classified as follows.

centrifugal governors.



We know that acceleration of the reciprocating parts.

$$A_R = A_P = \omega^2 r \left[\cos \theta + \frac{\cos 2\theta}{2} \right]$$

Accelerating force on main force of the reciprocating parts.

$$F = m \cdot a.$$

$$F_z = m R \cdot \omega^2 r \left[\cos \theta + \frac{\cos 2\theta}{2} \right]$$

It may be noted that in a horizontal engine the reciprocating parts are accelerated from left during the first half of the stroke (i.e. when the piston moves from inner dead centre to outer dead centre). It is then retarded during the latter half of the stroke (i.e. when the piston moves from outer dead centre to inner dead centre).

If P is the net pressure of steam on the piston and D is the diameter.

Net load in the piston

$$F_L = P_{mean} \times Area$$

$$= P \cdot \frac{\pi}{4} D^2$$

$$T = m(k_1)^2 \alpha \quad \text{--- (1)}$$

Similarly the torque required to accelerate the two-masses system placed arbitrarily.

$$T_1 = I_1 \alpha$$

$$T_1 = m(k_1)^2 \alpha \quad \text{--- (2)}$$

Difference between torque required to accelerate the two-mass system and the torque required to accelerate the rigid body.

$$T' = T_1 - T$$

$$T' = m k_1^2 \alpha - m k_1^2 \alpha$$

$$\boxed{T' = m \alpha [k_1^2 - k_1^2]} \quad \text{--- (3)}$$

The difference of torque T' is known as correction couple.

E

$$100 \times 250 = 25000$$

$$100 \times 250 = 25000$$

$$100 \times 250 = 25000$$

$$100 \times 250 = 25000$$

$$100 \times 250 = 25000$$

$$100 \times 250 = 25000$$

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$$100 \times 250 = 25000$$

$$100 \times 250 = 25000$$

Brake And Dynamometers.

Introduction.

A Brake is a device by means of which artificial frictional resistance is applied to a moving machine member, in order to retard or stop the motion of a machine. The capacity of brake depends upon the following factor.

The unit pressure b/w the braking surfaces.

The coefficient of friction b/w the braking surface.

The peripheral velocity of the brake drum.

The projected area of the friction surfaces.

The ability of the brake to dissipate heat equivalent to the energy being absorbed.

Materials for Brake lining.

It should have high coefficient of friction with minimum fading.

It should have low wear rate.

" " " high resistance.

" " " high dissipation capacity.

" " " adequate mechanical strength.

" should not be affected by moisture and oil.

3.

If $T =$ Torque on crankshaft at any instant,

$T_{mean} =$ mean twisting torque

Then acceleration torque on the rotating parts of the engine $= T - T_{mean}$.

4.

If $(T - T_{mean})$ is positive, the flywheel accelerates and if $(T - T_{mean})$ is negative, then the flywheel decelerates.

Note

Since $\omega = \frac{2\pi N}{60}$

Then eqn (2) becomes

$$\Delta E = I \omega (\omega_1 - \omega_2)$$

$$= I \times \frac{2\pi N}{60} \left[\frac{2\pi N_1}{60} - \frac{2\pi N_2}{60} \right]$$

$$= I \cdot \frac{4\pi^2}{3600} \times N [N_1 - N_2]$$

$$= \frac{4\pi^2}{3600} \times I N [N_1 - N_2]$$

$$= \frac{4\pi^2}{3600} (mk^2) \cdot N (N_1 - N_2)$$

$$= \frac{4\pi^2}{3600} mk^2 \cdot N^2 \cdot \frac{(N_1 - N_2)}{N}$$

$$\Delta E = \frac{4\pi^2}{3600} mk^2 \cdot N^2 \cdot C_s$$

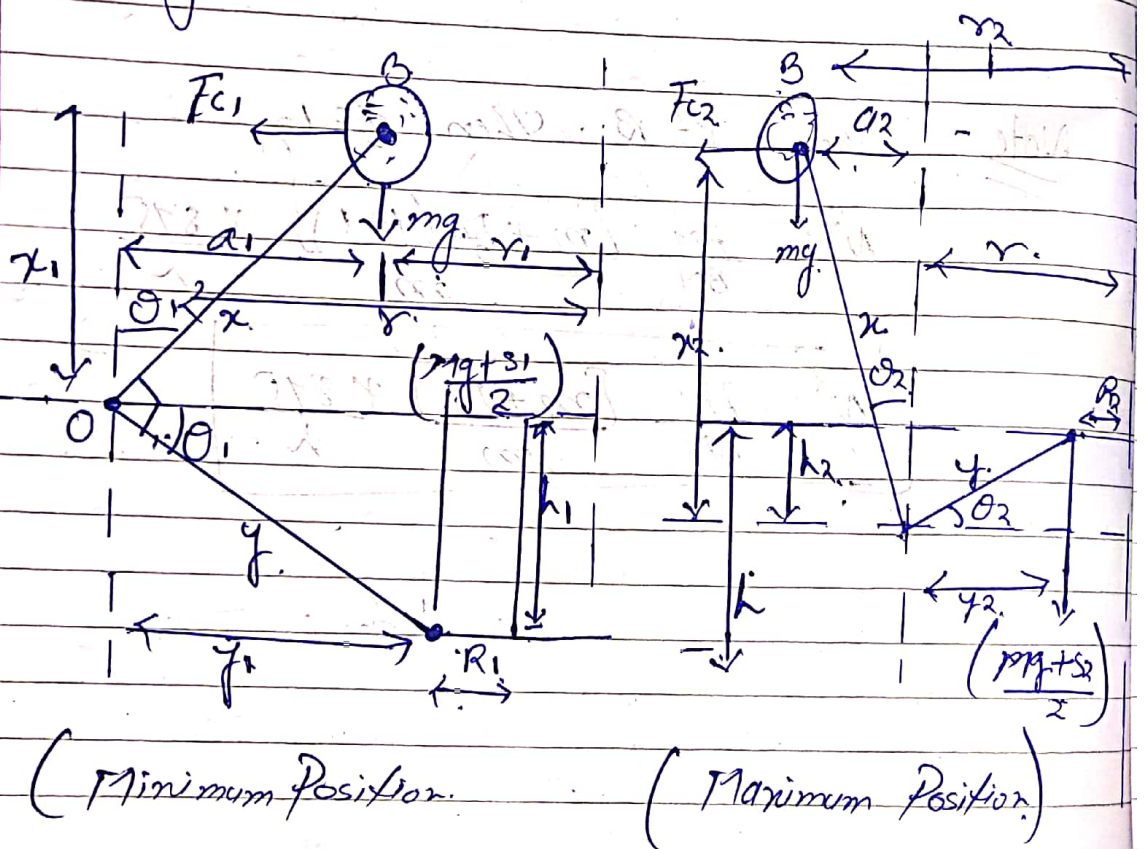
$$\therefore C_s = \frac{N_1 - N_2}{N}$$

$$\Delta E = \frac{\pi^2}{900} mk^2 \cdot N^2 \cdot C_s$$

A

Spring control governor (Hartnell governor)

It consists of two bell crank levers pivoted at the point O , O to the frame. The frame is attached to the governor spindle and therefore rotates with it. Each lever carries a ball at the end of the vertical arm OB and a roller at the end of the horizontal arm OR . A helical spring in compression provides equal downward forces on the two rollers through a collar on the sleeve.



Let m = mass of each ball in (kg)

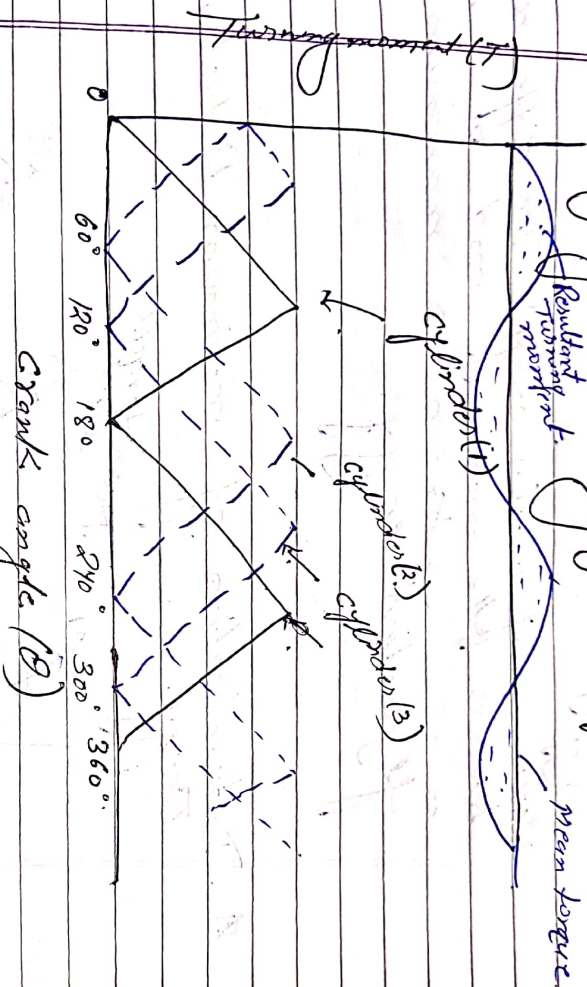
M = Mass of sleeve (kg)

r_1 = Minimum radius of rotation (m)

r_2 = Maximum radius of rotation (m)

on the piston is taken into account.

Turning moment diagram for a multi-cylinder engine.



A separate turning moment diagram for a compound steam engine having three cylinders (as the resultant turning moment diagram is shown in figure above). It may be noted that the first cylinder is the high pressure cylinder, the second cylinder is the intermediate cylinder and the third cylinder is the low pressure cylinder. The cranks are usually placed at 120° of each other.

Q Co-efficient of fluctuation of energy.
It may be defined as the ratio of the maximum fluctuation of energy to the work done per cycle.
co-efficient of fluctuation energy.

$$C_E = \frac{\text{Maximum fluctuation energy}}{\text{work done per cycle.}}$$

The work done per cycle is $(N-m \text{ or } T)$ may be obtained by using the following two relations.

$$1. \text{ work done per cycle} = T_{\text{mean}} \times \theta$$

where $T_{\text{mean}} = T_{\text{mean torque}}$

$\theta = \text{angle turned in one revolution}$
Note: θ is in rads of steam engine and two stroke I.C. engine.
 $= 4\pi$ in case of 4 stroke I.C. engine.

$$P(\text{Power}) = \frac{2\pi N}{60} T_{\text{mean}} \quad \{ P = W T \}$$

$$T_{\text{mean}} = \frac{60 \cdot P}{2\pi N}$$

$$= \frac{P}{\omega}$$

where

$P =$

$N =$

$\omega =$ angular speed in rad/s

Note

ϕ = inclination of crank connecting rod to the line of length connecting to the pin of crank
 $n = \frac{1}{r}$ and to the radius of crank

* velocity of the piston.

$$x = \frac{P'P}{P'O} = (PQ + OQ) \quad \text{--- (1)}$$

From triangle. $\triangle PQO$	From triangle $\triangle COQ$
$\cos \phi = \frac{PQ}{PO}$	$\cos \theta = \frac{OQ}{OC}$
$PQ = PO \cos \phi$	$OQ = OC \cos \theta$
$PQ = l \cos \phi$	$OQ = r \cos \theta$

Putting these values in (1)

$$\begin{aligned} &= (l + r) - (l \cos \phi + r \cos \theta) \\ &= l + r - l \cos \phi - r \cos \theta \\ &= l(1 - \cos \phi) + r(1 - \cos \theta) \end{aligned}$$

$$= r \left[(1 - \cos \theta) + \frac{1}{r} (1 - \cos \phi) \right]$$

But $\frac{1}{r} = n$ } Ratio of length of connecting rod to the radius.

$$= r \left[(1 - \cos \theta) + n(1 - \cos \phi) \right] \quad \text{--- (2)}$$

From $\triangle COQ$
 $\sin \phi =$
 $\cos =$

So CC

ing, so
PC, so
connecting
of circuit

From triangle CDB and COB

ΔCDB
 $\sin \theta = \frac{CB}{PC}$

$\sin \theta = \frac{CB}{r}$

$CB = r \sin \theta$

ΔCOB
 $\sin \theta = \frac{CB}{OC}$

$\sin \theta = \frac{CB}{r}$

$CB = r \sin \theta$

So $CB = r \sin \theta = r \sin \theta$

$\frac{1}{r} = \frac{\sin \theta}{\sin \phi}$

$n = \frac{\sin \theta}{\sin \phi}$

$\sin \phi = \frac{\sin \theta}{n}$

$\cos \phi = \sqrt{1 - \sin^2 \phi}$

$= \sqrt{1 - \frac{\sin^2 \theta}{n^2}}$

$\cos \phi = \left(1 - \frac{\sin^2 \theta}{n^2}\right)^{1/2}$

Applying binomial theorem.

$\cos \phi = 1 - \frac{1}{2} \cdot \frac{\sin^2 \theta}{n^2} + \dots$

$\cos \phi - 1 = -\frac{1}{2} \cdot \frac{\sin^2 \theta}{n^2}$

$1 - \cos \phi = \frac{1}{2} \cdot \frac{\sin^2 \theta}{n^2}$ — (3)

Date: 5-2-2020

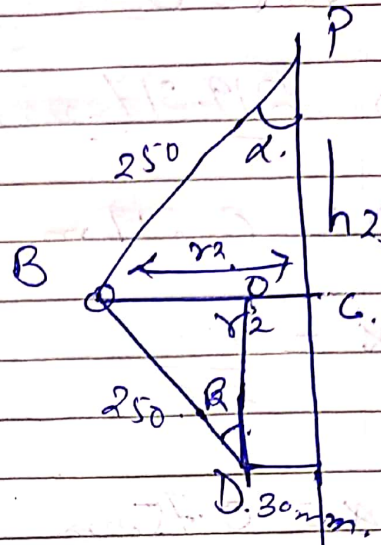
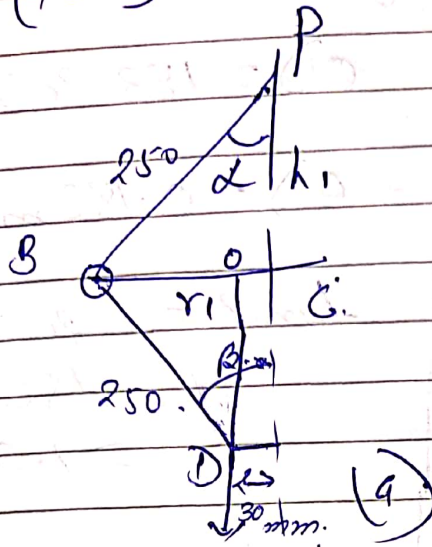
Questions of Governor

Page No.

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Date:

Example (18.5).



Given that: $R_1 = 150 \text{ mm}$
 $R_2 = 200 \text{ mm}$
 $m = 5 \text{ Kg}$
 $M = 50 \text{ Kg}$

$$(h_1)^2 = (250)^2 - (150)^2$$

$$h_1 = \sqrt{(250)^2 - (150)^2}$$

$$= \sqrt{40000}$$

$$h_1 = 200 \text{ mm}$$

$$h = 0.2 \text{ m}$$

$$BO = BC - OC$$

$$= 150 - 30$$

$$= 120 \text{ mm}$$

$$= 0.12 \text{ m}$$

From (a):

$$OD^2 = (250)^2 - (120)^2$$

$$OD = \sqrt{(250)^2 - (120)^2}$$

$$(h_2)^2 = (250)^2 - (200)^2$$

$$h_2 = \sqrt{(250)^2 - (200)^2}$$

$$h_2 = \sqrt{22500}$$

$$h_2 = 150 \text{ mm}$$

$$h_2 = 0.15 \text{ m}$$

$$BO = 200 - 30 \text{ mm}$$

$$BO = 170 \text{ mm}$$

$$= 0.17 \text{ m}$$

From (b):

$$OD^2 = (250)^2 - (170)^2$$

$$OD = \sqrt{(250)^2 - (170)^2}$$

$$\Delta E = \text{work done per cycle} \times CE$$

(ii)

(3) Given Power (P) = 300 kW
 $N = 90 \text{ r.p.m.}$
 $CE = 0.1$

$\left. \begin{array}{l} \text{CS not to be kept within } \pm 0.5\% \\ \text{trans speed} \end{array} \right\}$

$$K = 2 \text{ m}$$

$$\frac{\omega_1 - \omega_2}{\omega} = 1\%$$

$$CS = \frac{\omega_1 - \omega_2}{\omega} = 0.01$$

$$\text{Work done per cycle} = \frac{P \times 60}{N}$$

$$= \frac{300 \times 10^3 \times 60}{8 \times 90} = 200 \times 10^3 \text{ N-m}$$

$$\Delta E = \text{work done per cycle} \times CE$$

$$= 200 \times 10^3 \times 0.1$$

$$= 20 \times 10^3 \text{ N-m}$$

$$\Delta E = \frac{m}{900} \text{ m}^2 \cdot \text{K}^2 \cdot \text{N}^2 \cdot \text{CS}$$

$$m = \frac{\Delta E \times 900}{\text{m}^2 \cdot \text{K}^2 \cdot \text{N}^2 \cdot \text{CS}}$$

$$= \frac{20 \times 10^3 \times 900}{\text{m}^2 \times (2)^2 \times (90)^2 \times 0.01}$$

$$m = 5628.95 \text{ kg}$$

$$= 5629 \text{ kg}$$

Ex 1.1 Given
 $N = 60 \text{ r.p.m}$
 $N_2 = 61 \text{ r.p.m}$

By Using Formula:

$$h = \frac{895}{N^2}$$

$$h_1 = \frac{895}{(60)^2} = 0.2486 \text{ m.}$$

$$h_2 = \frac{895}{(61)^2} = 0.24052 \text{ m.}$$

Change in verticle height.

$$= h_1 - h_2$$

$$= 0.2486 - 0.24052$$

$$= 8.08 \times 10^{-3}$$

$$= 0.008 \text{ m.}$$

$$0.008 \times 10^3 \text{ mm.}$$

$$= 8 \text{ mm.}$$

on Solving (1) & (2)

$$\begin{aligned} N_1 - N_2 &= 2 \\ N_1 + N_2 &= 240 \end{aligned}$$

$$-2N_2 = -238$$

$$N_2 = \frac{238}{2} = 119 \text{ r.p.m.}$$

$$\begin{aligned} \text{So } N_1 &= 2 + N_2 \\ &= 2 + 119 \\ &= 121 \text{ r.p.m.} \end{aligned}$$

Given

$$\begin{aligned} (2) \quad K &= 1 \text{ m} \\ m &= 2500 \text{ kg} \end{aligned}$$

$$T = 1500 \text{ N-m}$$

Note

$$\text{Torque} = I \alpha$$

$$\text{where } I = m \cdot k^2$$

$$\alpha = \text{angular acceleration}$$

$$\begin{aligned} (1) \quad I &= m k^2 \\ &= (1)^2 (2500) \\ &= 2500 \text{ kg-m}^2 \end{aligned}$$

$$T = I \alpha$$

$$\alpha = \frac{T}{I} = \frac{1500}{2500} = 0.6 \text{ rad/sec}$$

For

Applying Pythagoras Theorem in $\triangle BCD$

$$BD^2 = BC^2 + CD^2 \quad \text{By (b)}$$

$$DC^2 = BD^2 - BC^2$$

$$DC = \sqrt{BD^2 - BC^2}$$

$$CD = \sqrt{BD^2 - BC^2}$$

$$= \sqrt{(950)^2 - (900)^2}$$

$$= \sqrt{52500}$$

$$= \sqrt{(950)^2 - (128.55)^2}$$

$$CD = 229.12$$

$$CD = 0.23 \text{ m}$$

$$DC = 45974.89$$

$$DC = 214.41 \text{ m}$$

$$D = 0.214 \text{ m}$$

$$\tan B_1 = \frac{0.1}{0.23}$$

$$\tan B_2 = \frac{0.128}{0.214}$$

$$\tan B_1 = 0.4347$$

$$\tan B_2 = 0.5981$$

$$q = \frac{\tan B_1}{\tan B_2}$$

$$q = \frac{\tan B_2}{\tan B_1}$$

$$= \frac{0.4348}{0.5774}$$

$$= 0.5981$$

$$q = 0.7530$$

$$= 0.83109$$

$$q_2 = 0.71279$$

$$N^2 = \frac{m + \frac{M}{2}(1+e)}{m} \times \frac{g}{h} \left(\frac{60}{2\pi} \right)^2$$

$$N^2 = \frac{m + \frac{M}{2}(1+e)}{m} \times \frac{895}{h}$$

$$N^2 = \frac{m + \frac{M}{2}(1+e)}{m} \times \frac{895}{h}$$

if $e=1$

$$N^2 = \left(\frac{m+M}{m} \right) \frac{895}{h}$$

Ans

In case of friction Force.

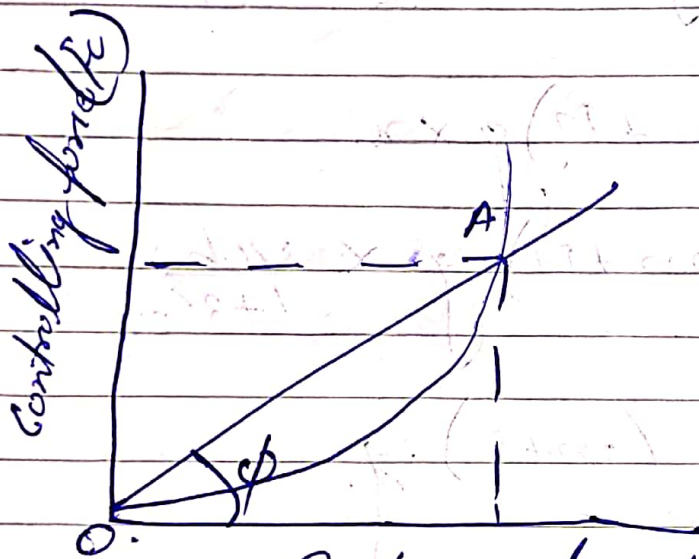
$$N^2 = \frac{mg + \left[\frac{Mg}{2} + F \right] (1+e)}{mg} \times \frac{895}{h}$$

A Controlling force:

In case of a governor running at a steady speed the inward force acting on the rotating ball is known as controlling force. It is equal to the and opposite to the centrifugal reaction.

controlling force

$$F_c = m \cdot \omega^2 \cdot r$$



Radius of rotation (r) $\rightarrow$

when the graph b/w the controlling force F_c and radius of rotation of ball (r) is drawn the graph obtained is known as controlling force diagram.

$$(N_2)^2 = 56566.9883$$

$$(N_2) = 237.838$$

$$= 238 \text{ r.p.m.}$$

Range of speed

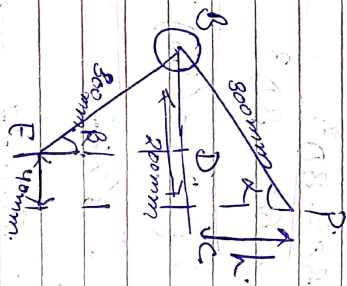
$$= N_2 - N_1$$

$$= 238 - 208$$

$$= 30 \text{ r.p.m.}$$

Ans

Ex 18.6



Given: $m = 10 \text{ kg}$

$\mu = 70 \text{ kg}$

$F = 20 \text{ N}$

Ans

BC = 200 mm

DD = 160 mm

$$R_N \times x = P \times l$$

$$R_N = \frac{P \times l}{x}$$

Braking torque.

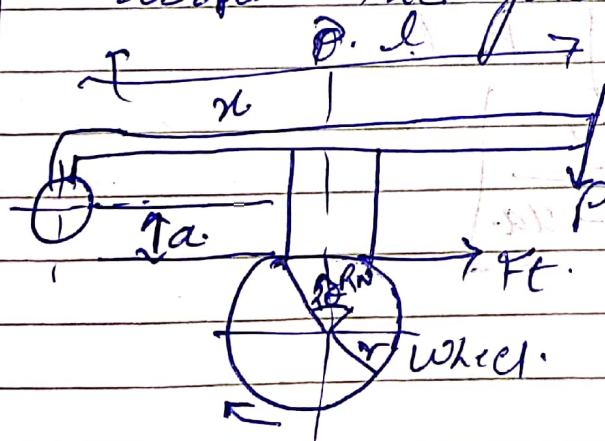
$$T_B = \mu \cdot R_N \times r$$

$$T_B = \mu \cdot \frac{P \times l}{x} \times r = \frac{\mu \cdot P \cdot l \cdot r}{x}$$

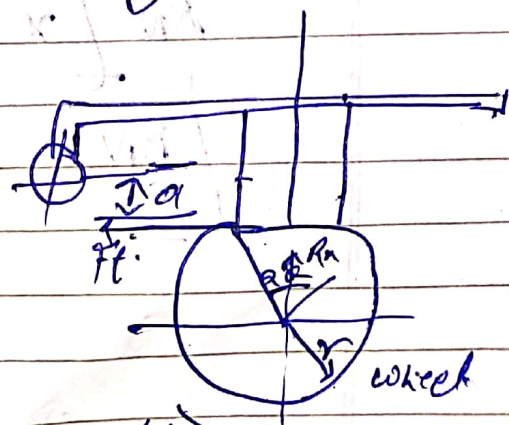
It may be noted that when the braking wheel rotates anticlockwise in shown in figure (b) then the braking torque is same.

$$T_B = \mu \cdot R_N \cdot r = \frac{\mu \cdot P \cdot l \cdot r}{x}$$

Case (IInd). When the line of action of the tangential braking force (F_t) passes through a distance (a) below the fulcrum O and the brake wheel rotates clockwise as in shown in figure (a) then for equilibrium taking moment about the fulcrum O .



(a) clockwise rotation



(b) anticlockwise

From (A) & (B)

$$\cancel{2} \left(\frac{F_{C2} - F_{C1}}{r_2 - r} \right) \left(\frac{r}{\cancel{2}} \right)^3 = \cancel{2} \left(\frac{F_C - F_{C1}}{r - r_1} \right) \left(\frac{r}{\cancel{2}} \right)^3$$

$$\left(\frac{F_{C2} - F_{C1}}{r_2 - r_1} \right) = \left(\frac{F_C - F_{C1}}{r - r_1} \right)$$

$$\frac{F_{C2} - F_C}{r_2 - r_1} = \frac{F_C - F_{C1}}{r - r_1}$$

$$\frac{F_{C2} + F_{C1}}{r_2 - r_1} = \frac{F_C + F_C}{r - r_1}$$

$$q = \frac{\tan B}{\tan \alpha}$$

$$= \frac{0.843}{0.78811} = 1.06996$$

$$= \frac{0.15811}{0.8298} = 0.19054$$

$$= \frac{0.6304}{0.8298} = 0.7600$$

$$(N)^2 = \frac{m + \frac{1}{2}(1+q)}{m} \times \frac{895}{k}$$

$$= \frac{10 + \frac{70}{2}(1+0.7060)}{10} \times \frac{895}{0.223}$$

$$(N)^2 = 27977.780$$

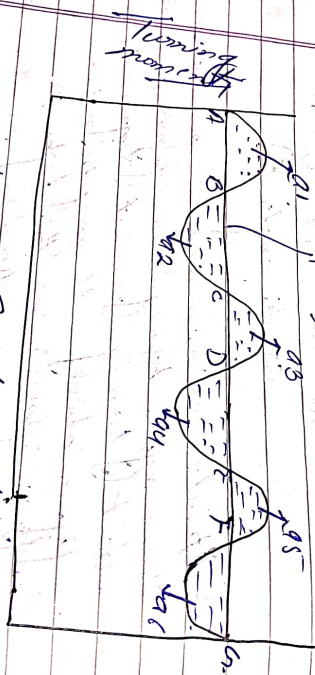
$$N = 167.26 \text{ p.m.}$$

Given Friction force $F = 20 \text{ N}$.

$$(N) = \frac{mg \left(\frac{14}{2} - N \right) (1+q) \times \frac{895}{k}}{m}$$

4 The variation of fluctuation energy below the mean energy line are called fluctuation energy. The difference between the maximum and minimum energies is known as maximum fluctuation energy.

5 Determination of maximum fluctuation energy.



Crank angle

The turning moment diagram for multi-cylinder engine is shown by a curve. The horizontal line is the mean torque line and the area above the

The area below the mean torque line these area represent some quantity of energy which is either added or subtracted from the input of the moving parts of the engine.

Let the energy in the flywheel at A = E

Energy at B = E + a₁

" " at C = E + a₁ - a₂

" " D = E + a₁ - a₂ + a₃

" " E = E + a₁ - a₂ + a₃ - a₄

" " F = E + a₁ - a₂ + a₃ - a₄ + a₅

" " G = E + a₁ - a₂ + a₃ - a₄ + a₅ - a₆

= Energy at A (i.e. cycle repeats after 6a)

Let us know suppose that the greatest of these energies is at B and least at E. Therefore

Maximum energy in flywheel

$$= E + a_1$$

Minimum energy in the flywheel

$$= E + a_1 - a_2 + a_3 - a_4$$

Maximum fluctuation Energy

$$\Delta E = \text{Maximum Energy} - \text{Minimum}$$

$$= a_2 - a_3 + a_4$$

Now consider the case of Hartnell governor running at N_1 and N_2 .

$$M_1 g + S_1 = 2 F C_1 \cdot \frac{r_1}{r_2}$$

$$= 2 m r \omega_1^2 \cdot \frac{r_1}{r_2}$$

$$= 2 m \left(\frac{2\pi N_1}{60} \right)^2 \cdot r_1 \times \frac{r_1}{r_2} \quad \text{--- (1)}$$

$$M_1 g + S_2 = 2 F C_2 \cdot \frac{r_2}{r_1}$$

$$= 2 m \left(\frac{2\pi N_2}{60} \right)^2 \cdot r_2 \times \frac{r_2}{r_1} \quad \text{--- (2)}$$

For isochronous $N_1 = N_2$.

$$\frac{M_1 g + S_1}{M_2 g + S_2} = \frac{r_1}{r_2}$$

Note — The isochronous governor is not of practical use because the sleeve will move to one of its extreme positions immediately the speed deviates from the isochronous speed.

A Hinting:

A governor is to be hunt if its speed of the engine fluctuates continuously above and below the mean speed. This is caused by a too sensitive governor will changes the fuel supply by a large amount when a small change in the speed of rotation produces large

and resolving F_B along the crank

$$\sin [90^\circ - (\theta + \phi)] = \frac{F_B}{F_Q}$$

$$\cos (\theta + \phi) = \frac{F_B}{F_Q}$$

$$F_B = F_Q \cdot \cos (\theta + \phi)$$

$$F_B = \frac{F_P}{\cos \phi} \cos (\theta + \phi)$$

(5) Crank effort or turning moment or torque on the crank shaft:

The product of the crank pin effort (F_P) and crank pin radius (r) is known as crank effort or turning moment or torque on the crank shaft.

∴ crank effort

$$T = F_P \times r$$

$$= \frac{F_P}{\cos \phi} \sin (\theta + \phi) \times r$$

$$= \frac{F_P}{\cos \phi} [\sin \theta \cdot \cos \phi + \cos \theta \sin \phi] \times r$$

$$= F_P [\sin \theta + \cos \theta \cdot \tan \phi] \times r$$

(1)

$$\frac{m \cancel{x} \omega^2}{h} = \cancel{mg} + \frac{mg}{2} [1+q]$$

$$m h \omega^2 = mg + \frac{mg}{2} [1+q]$$

$$h = \frac{mg + \frac{mg}{2} [1+q]}{m} \times \frac{1}{\omega^2}$$

* In case of $\tan R = \tan d =$
i.e. $q = 1$

$$h = \frac{mg + \frac{mg}{2} [1+1]}{m} \times \frac{1}{\omega^2}$$

$$h = \frac{g(m+1)}{m \omega^2}$$

But we know that

$$l \sin \phi = r \sin \theta$$

$$\frac{l}{r} = \frac{\sin \theta}{\sin \phi}$$

$$\sin \phi = \frac{r}{l} \sin \theta$$

$$\sin \phi = \frac{\sin \theta}{n} \quad \text{--- (A)}$$

$$\cos \phi = \sqrt{1 - \frac{\sin^2 \theta}{n^2}}$$

$$= \sqrt{\frac{n^2 - \sin^2 \theta}{n^2}}$$

$$\cos \phi = \frac{\sqrt{n^2 - \sin^2 \theta}}{n} \quad \text{--- (B)}$$

$$\tan \phi = \frac{\sin \phi}{\cos \phi} = \frac{\sin \theta \times r}{n \sqrt{n^2 - \sin^2 \theta}} \quad \left\{ \text{from A \& B} \right\}$$

$$\tan \phi = \frac{\sin \theta}{\sqrt{n^2 - \sin^2 \theta}}$$

Substitute $\tan \phi$ in (1)

$$= \mu \times r \left[\sin \theta + \cos \theta \cdot \frac{\sin \theta}{\sqrt{n^2 - \sin^2 \theta}} \right]$$

$$= \mu \times r \left[\sin \theta + \frac{1}{2} \frac{\sin 2\theta}{\sqrt{n^2 - \sin^2 \theta}} \right]$$

$$= \frac{1}{2} I [\omega_1^2 - \omega_2^2]$$

$$= \frac{1}{2} I (\omega_1 - \omega_2) (\omega_1 + \omega_2) \quad \text{--- (1)}$$

$$\left\{ \text{But } \frac{\omega_1 + \omega_2}{2} = \omega \right\}$$

$$= I (\omega_1 - \omega_2) \omega \quad \text{--- (2)}$$

$$\Delta E = I \frac{(\omega_1 - \omega_2)}{\omega} \omega^2$$

$$\therefore \left\{ \frac{\omega_1 - \omega_2}{2\omega} = c_s \right\}$$

$$\Delta E = I \cdot \omega^2 \cdot c_s$$

$$= m k^2 \cdot \omega^2 \cdot c_s$$

$$\Delta E = 2 E \cdot c_s \quad \text{N-m or J}$$

$$\left\{ \begin{array}{l} I = m k^2 \\ E = \frac{1}{2} I \omega^2 \end{array} \right\}$$

The radius of gyration (k) may be taken equal to the mean radius of the rim (R).

$$\Delta E = m R^2 \cdot \omega^2 \cdot c_s$$

$$\Delta E = m v^2 \cdot c_s$$

Where v = mean linear velocity ($R\omega$)

Neglecting c^2 because very small.

$$= 2h \left(\frac{2c}{1+c^2} \right)$$

$$x = \frac{4ch}{(1+c^2)} = \frac{4ch}{1+2c}$$

Power of governor = Effort $\times$ stevelift

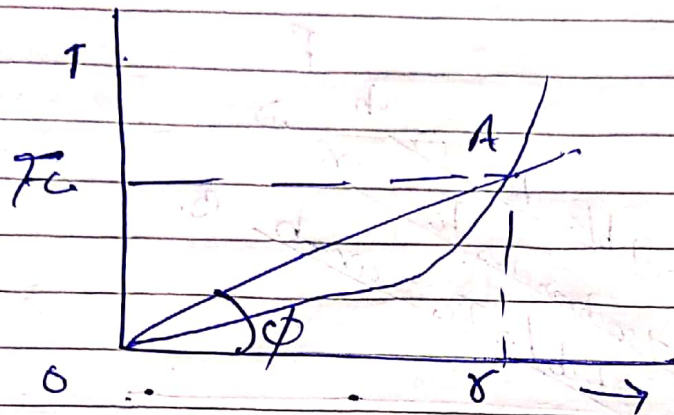
$$= C \cdot (m+r)g \times x$$

$$= C \cdot (m+r)g \times \frac{4ch}{1+2c}$$

$$= \frac{4c^2h}{1+2c} (m+r)g$$

$$P = \frac{4c^2}{1+2c} (m+r)g \cdot h$$

Q. Controlling force diagram for Porter governor.



$$F_c = m \cdot r \cdot \omega^2$$

$$= m \cdot r \left(\frac{2\pi N}{60} \right)^2$$

$$N^2 = \frac{(60)^2 \cdot F_c}{m \cdot r \cdot (2\pi)^2}$$

$$\text{But } \frac{F_c}{r} = \tan \phi$$

$$N^2 = \frac{1}{m} \left(\frac{60}{2\pi} \right)^2 \cdot \tan \phi$$

$$N = \frac{60}{2\pi} \left(\frac{\tan \phi}{m} \right)^{1/2}$$

Note:

The equilibrium speed must increase with the increase of radius of rotation of the governor balls.

Brake

Types of Brake
 1. Hydraulic brakes
 2. Pumps or fluid brakes
 3. Electric brakes
 4. Mechanical brakes
 5. Air brakes

The mechanical brakes according to the direction of acting force may be divided into the following two groups

(1) Radial brakes: In these brakes the force acting on the brake drum is in radial direction. The radial brakes may be sub-divided into external and internal brake. According to the shape of the friction element these brakes may be block or shoe brakes and band brakes.

(2) Axial brakes: In these brakes the force acting on the brake drum is in axial direction. The axial brake may be disc brake and cone brakes.

==

Governor Power = Effort $\times$ sleeve lift

$$P = E \times x$$

$$\therefore x = 2(h - h_1) \quad \text{--- (A)}$$

$$h = \frac{m + M}{m} \times \frac{895}{N^2}$$

$$h_1 = \frac{m + M_0}{m} \times \frac{895}{(1+c)^2 N^2}$$

$$\text{So } \frac{h_1}{h} = \frac{\frac{m + M_0}{m} \times \frac{1}{(1+c)^2}}{\frac{m + M}{m} \times \frac{1}{(1+c)^2}}$$

$$\frac{h_1}{h} = \frac{1}{(1+c)^2}$$

$$h_1 = \frac{h}{(1+c)^2}$$

Substitute h_1 in (A)

$$x = 2 \left[h - \frac{h}{(1+c)^2} \right]$$

$$= 2h \left[\frac{(1+c)^2 - 1}{(1+c)^2} \right]$$

$$= 2h \left[\frac{(1+c)^2 - 1}{(1+c)^2} \right]$$

$$= 2h \left[\frac{1 + c^2 + 2c - 1}{(1+c)^2} \right]$$

20/11/20

Unit - Ist (2nd Part)

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* Turning moment diagram and fly wheel.

Introduction: The turning moment diagram (also known as crank effort diagram) is the graphical representation of the turning moment or crank effort for various positions of the crank. It is plotted on Cartesian co-ordinates in which the turning moment is taken as the ordinate and crank angle as abscissa.

* Turning moment diagram for a single cylinder double acting steam engine.

As shown in the figure. The vertical ordinate represent the turning moment and the horizontal ordinate represent crank angle.

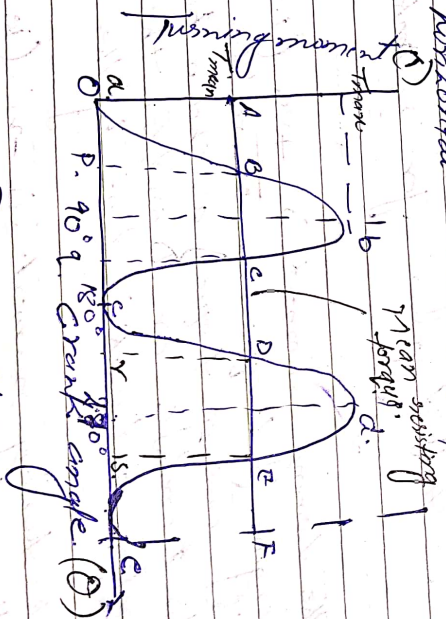


Fig - 1 Piston effort

2 B in 3

$$m_k^3 = \frac{m_2 (1)}{(1+12)} + \frac{m_1 (12)}{(1+12)}$$

$$p_k \cdot k_0 = p_{k+1} \cdot k_2 \int_{(1+x)^2}^{1+x} (1+x) dx$$

$$h_0 = 1.17$$

Printed block and shoe brake.

We have discussed in the previous article that even when the angle of contact is less than 60° then it may be assumed that the normal pressure between the block and the ~~area~~ wheel is uniform. But when the angle of contact is greater than 60° ,

Effort and Power of a Governor

* Effort of a Governor: The effort of a governor is the mean force exerted at a sleeve for a given percentage change of speed. (or lift of the sleeve). It may be noted that within the governor's running stability, there is no force at the sleeve. But when the speed changes there is a resistance at the sleeve which opposes its motion. It is assumed that this resistance which is equal to the effort, varies inversely from a minimum value to zero. As the governor moves into its new position of equilibrium.

* The Power of a governor: It is the work done at the sleeve for a given percentage change of speed. It is the product of the upward value of the effort and the distance through which the sleeve moves.

$$\text{Power} = \text{Mean effort} \times \text{Lift of sleeve.}$$

$$BD^2 = PC^2 + BC^2$$

$$PC^2 = BD^2 - BC^2$$

$$PC = \sqrt{BD^2 - BC^2}$$

$$= \sqrt{(300)^2 - (200)^2}$$

$$= \sqrt{50000}$$

$$PC = 223.60 \text{ mm}$$

$$PC = 0.223 \text{ m}$$

$$BC^2 = BD^2 + ED^2$$

$$ED^2 = BC^2 - BD^2$$

$$ED = \sqrt{BC^2 - BD^2}$$

$$= \sqrt{(300)^2 - (160)^2}$$

$$ED = \sqrt{84400}$$

$$ED = 290.77 \text{ mm}$$

$$= 0.293 \text{ m}$$

$$\tan \alpha = \frac{200}{223.60}$$

$$= 0.893$$

$$\tan \beta = \frac{200}{253.77}$$

$$= 0.78881$$

$$\tan R = \frac{160}{253.77}$$

$$= 0.6304$$

(2)

$$(W)^2 = \frac{10 + (70 - 20)}{2} \left(\frac{1 + 0.7060}{2} \right) \times \frac{875}{0.223}$$

$$(W)^2 = \frac{2180 \cdot 875}{2}$$

$$W = 145.3644 \text{ m}$$

$$27279.11$$

$$= \frac{10 \times (9.8) + \frac{(70 \cdot (9.8) - 20)}{2} (1 + 0.7060)}{2} \times \frac{875}{0.223}$$

$$= \frac{10 \times 9.8}{2}$$

$$(N_1)^2 = 27279.11 \text{ v.p.m.}$$

$$N_1 = 165.185.16 \text{ v.p.m.}$$

$$(N_2)^2 = \frac{m \cdot g + \left(\frac{m \cdot g + 20}{2} \right) (1 + q)}{10 \cdot m \cdot g} \times \frac{875}{2}$$

$$= \frac{10 \cdot (9.8) + \left(\frac{70 \cdot (9.8) + 20}{2} \right) (1 + 0.7060)}{10 \times 10 \cdot (9.8)} \times \frac{875}{0.223}$$

$$(N_2)^2 = 28676.44871$$

$$18.7]$$

$$OD = 14810$$

$$OD = \frac{219.517 \text{ mm}}{14}$$

$$OD = 0.219 \text{ m}$$

$$\tan \alpha = \frac{150}{200}$$

$$\tan \alpha = 0.75$$

$$\tan \beta = \frac{120}{219} = 0.5479$$

$$= 0.5479$$

$$q = \frac{\tan \beta}{\tan \alpha} = \frac{0.5479}{0.75}$$

$$q = 0.7305$$

$$(N_1)^3 = \frac{m \cdot g + \frac{1}{2} (1+q) \times 845}{\frac{m \cdot g}{h_1}}$$

$$= \frac{5 \times 9.81 + \frac{1}{2} (1+0.7305) \times 845}{5 \times 9.81}$$

$$(N_1)^3 = \frac{5 \times 9.81 + 50 \times 9.81 (1+0.7305)}{5 \times 9.81}$$

$$= 1.3333$$

$$(N_1)^2 = 4313.75$$

$$N_1 = 207.807 \text{ m.p.m}$$

$$OD = 183.303 \text{ mm}$$

$$OD = 0.183 \text{ m}$$

$$\tan \alpha = \frac{200}{150}$$

$$= 1.3333$$

$$\tan \beta = \frac{170}{123} = 0.723$$

$$q = \frac{\tan \beta}{\tan \alpha}$$

$$= \frac{0.723}{1.3333}$$

$$q_2 = 0.6761$$

$$(N_2)^3 = \frac{m \cdot g + \frac{1}{2} (1+q) \times 845}{\frac{m \cdot g}{h_2}}$$

$$= \frac{5 \times 9.81 + \frac{1}{2} (1+0.6761) \times 845}{5 \times 9.81}$$

$$(N_2)^3 = \frac{5 \times 9.81 + 50 \times 9.81 (1+0.6761)}{5 \times 9.81}$$

$$= 1.3333$$

$$N_2 = 207.807 \text{ m.p.m}$$

$$(N_2)^2$$

$$(N_2)$$

$$P_{av}$$

$$[5 \times 18.6]$$

(2)

Force acting along the connecting rod. denoted by F_{13} . It is denoted by F_{13} .

$$F_{13} \cos \phi = \frac{\text{base}}{\text{hypotenuse}}$$

$$\cos \phi = \frac{F_{13}}{F_{13}}$$

$$F_{13} = \frac{F_P}{\cos \phi}$$

$$\text{but } \cos \phi = \sqrt{1 - \frac{\sin^2 \theta}{\eta^2}}$$

$$F_{13} = \frac{F_P}{\sqrt{1 - \frac{\sin^2 \theta}{\eta^2}}}$$

(3)

Thrust on the sides of the cylinder walls or normal reaction on the guide bars.

It is denoted by F_N

$$\sin \phi = \frac{P}{h}$$

$$\sin \phi = \frac{F_N}{F_{13}}$$

$$F_N = F_{13} \cdot \sin \phi$$

But

So

Cx

Th

px

cx

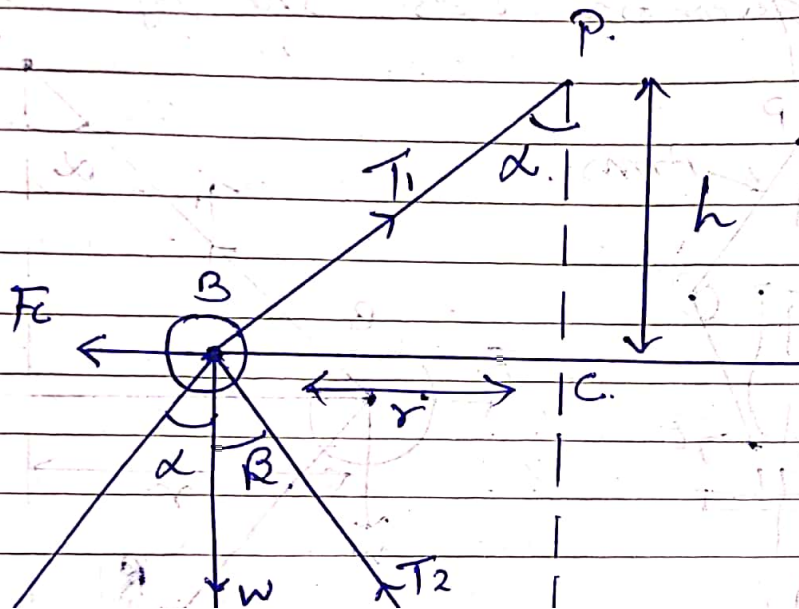
o

$\alpha =$ Angle of inclination of the arm (or upper link) to the vertical.

$\beta =$ Angle of inclination of the link (lower link) to the vertical.

Two method (i) method of resolving forces
(ii) Instantaneous centre method.

* By Instantaneous centre method.



Speed
Energy

Note

The
fluc
it

Q Co-efficient of fluctuation
The difference between the maximum and minimum speed during a cycle is called the maximum fluctuation of speed. The ratio of maximum fluctuation of speed to the mean speed is called the Co-efficient of fluctuation of speed.

Let N_1 and N_2 = maximum and minimum speed in r.p.m during cycle.

$$N = \text{Mean Speed in r.p.m} = \frac{N_1 + N_2}{2}$$

Co-efficient of fluctuation of speed.

$$C_s = \frac{N_1 - N_2}{2N}$$

$$= \frac{N_1 - N_2}{\left(\frac{N_1 + N_2}{2} \right)} = \boxed{\frac{2(N_1 - N_2)}{N_1 + N_2}}$$

or
In case of angular speed.

$$= \frac{\omega_1 - \omega_2}{\frac{\omega_1 + \omega_2}{2}} = \boxed{\frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2}}$$

or
In case of linear speed.

$$= \frac{V_1 - V_2}{\frac{V_1 + V_2}{2}} = \boxed{\frac{2(V_1 - V_2)}{V_1 + V_2}}$$

$$F_c \times BM = mg \times IM + \frac{r_1 g}{2} ID$$

$$F_c = mg \cdot \frac{IM}{BM} + \frac{r_1 g}{2} \frac{ID}{BM}$$

$$= mg \cdot \frac{IM}{BM} + \frac{r_1 g}{2} \left[\frac{IM + ID}{BM} \right]$$

$$= mg \cdot \frac{IM}{BM} + \frac{r_1 g}{2} \left[\frac{IM}{BM} + \frac{ID}{BM} \right]$$

$$\left. \begin{array}{l} \text{In triangle } IBM, \text{ In triangle } DBM \\ \tan \alpha = \frac{IM}{BM}, \quad \frac{r_1 ID}{BM} = \tan B \end{array} \right\}$$

$$F_c = mg \tan \alpha + \frac{r_1 g}{2} [\tan \alpha + \tan B]$$

Dividing both sides by $\tan \alpha$.

$$\frac{F_c}{\tan \alpha} = mg + \frac{r_1 g}{2} \left[1 + \frac{\tan B}{\tan \alpha} \right]$$

$$\frac{F_c}{\tan \alpha} = mg + \frac{r_1 g}{2} [1 + q] \quad \text{--- (1)}$$

$$\therefore \text{where } q = \frac{\tan B}{\tan \alpha}$$

$$\text{But } F_c = m r \omega^2$$

$$\tan \alpha = \frac{r}{h} \quad \dots \quad \left. \begin{array}{l} \text{From } \Delta PBC \end{array} \right\}$$

Substituting in (1)

Substituting eqn (3) in (2)

$$x = r \left[(1 - \cos \theta) + n \left(\frac{1}{2} \frac{\sin^2 \theta}{n} \right) \right]$$

$$x = r \left(1 - \cos \theta \right) + \left(\frac{1}{2} \frac{\sin^2 \theta}{n} \right)$$

Diff. with respect to θ

$$\frac{dx}{d\theta} = r [0 - (-\sin \theta)] + \frac{1}{2n} 2 \sin \theta \cdot \cos \theta$$

$$= r \sin \theta + \frac{1}{2n} 2 \sin \theta \cdot \cos \theta$$

$$\frac{dx}{d\theta} = r \sin \theta + \frac{1}{2n} \sin 2\theta$$

* Velocity of the Piston P

$$v_p = \frac{dx}{d\theta} \cdot \frac{d\theta}{dt}$$

$$v_p = r \sin \theta + \frac{1}{2n} \sin 2\theta \cdot \omega$$

$$\left\{ \frac{d\theta}{dt} = \omega \right\}$$

$$v_p = \omega r \left[\sin \theta + \frac{\sin 2\theta}{2n} \right] \quad \text{--- (4)}$$

Acceleration of Piston

Diff. equation (4) w.r.t θ

$$\frac{dv}{d\theta} = \omega r \left[\cos \theta + \frac{1}{2n} \cos 2\theta \cdot 2 \right]$$

$$= \omega r \left[\cos \theta + \frac{\cos 2\theta}{n} \right]$$

Turning moment on crank shaft.

$$T = F_B \times r \left[\sin \theta + \frac{\sin 2\theta}{2} \right]$$

- F_B = Piston effort of crank shaft.
- r = Radius of connecting rod large end.
- r_1 = Ratio of radius of crank and radius of connecting rod.
- θ = Angle of turned by the crank from inner dead centre.

From diagram turning moment T is zero at angle $(\theta) = 0^\circ$ and 180° .
 when crank angle is 90° and 270° is again zero when crank angle is 180° .

The curve abc is represent the turning moment diagram for outstroke. The curve cde is that diagram for instroke.

Note

1. When the turning moment is +ve (i.e. when the engine torque is more than the mean resisting torque) as shown between Points B and C or D and E, the cranks accelerate and work is done by the steam.
2. When the turning moment is -ve (i.e. engine mean resisting torque is less than the cranks and work is done on the steam).

When the turning moment is -ve (i.e. engine mean resisting torque is less than the cranks and work is done on the steam).

Let:

 $m =$ Mass of the ball in kg. $w =$ weight of the ball in newtons $= m \cdot g$. $T =$ Tension in the arm in newtons. $\omega =$ Angular velocity of the arm and ball (rad/sec). $r =$ Radius of the path of rotation of the ball i.e. (horizontal) distance. $F_c =$ Centrifugal force acting on the ball in newtons $= m \omega^2 r$. $h =$ Height of governor in meter.

It is assumed that the weight of the arms, links, and the sleeve are negligible as compared to the weight of the ball.

In equilibrium condition.

Taking moment about point O, (From (1)).

$$F_c \times h = w \times r$$

$$F_c \times h = m \cdot g \cdot r$$

$$h = \frac{m g r}{F_c}$$

$$\therefore F_c = m r \omega^2$$

$$= \frac{r h g r}{m r \omega^2}$$

$$\boxed{F_c = \frac{g}{\omega^2}}$$

$$\text{Also, } h = \frac{g}{\omega^2}$$

$$h = \frac{g}{\left(\frac{2\pi N}{60}\right)^2}$$

$$h = \frac{9.81 \times 3600}{(2\pi N)^2}$$

$$\boxed{h = \frac{895}{N^2}}$$

But

$$1 \text{ mm} = 600 \text{ N-m.}$$

$$1 \text{ mm} = 3^\circ$$

$$= \frac{8 \times 14}{180} \times \frac{1000}{100} = 31.41 \text{ N-m.}$$

So $\Delta E = 172 \times 31.41$

$$= 5403.53 \text{ N-m.}$$

also given:

$\{ \text{CS. mol exceed to } \pm 1.5\%$

$$\frac{\omega_1 - \omega_2}{\omega} = 3\%$$

$$\omega_1 - \omega_2 = 0.03 \omega$$

$$\text{CS} = \frac{\omega_1 - \omega_2}{2\omega} = 0.03.$$

$$\Delta E = m R^3 \cdot \omega^3 \cdot \text{CS}$$

$$5403.53 = m \cdot (0.5)^3 \cdot \left(\frac{2\pi(600)}{60} \right)^3 \times 0.03$$

$$5403.53 = m (0.5)^3 \times (3947.84) \times 0.03$$

$$m = \frac{5403.53}{(0.5)^3 \times (3947.84) \times 0.03}$$

$$m = 182.4974 \text{ kg.}$$

$$= 183 \text{ kg.}$$

Consider a rigid body having its centre of gravity at G .
Let m = mass of the body about G .
 r = radius of gyration about G .
 r^2 = radius of gyration about G .

m_1, m_2 = Two masses from G .
 d_1 = Distance of mass m_1 from G .
 d_2 = Distance of mass m_2 from G .
 L = Total distance b/w m_1 & m_2 .

Thus, for the two masses to be dynamically equivalent

$$m_1 + m_2 = m \quad \text{--- (1)}$$

$$m_1 d_1 = m d_2 \quad \text{--- (2)}$$

$$m_1 (d_1)^2 = m_2 (d_2)^2 + m_2 (L_2)^2 \quad \text{--- (3)}$$

From equation (1) & (2)

$$m_1 = \frac{m d_2}{d_1}$$

$$m = \frac{m d_2}{d_1} + m_2$$

$$m = \frac{m d_2 + m_2 d_1}{d_1}$$

$$m d_1 = m_2 (d_1 + d_2)$$

$$m_2 = \frac{m d_1}{(d_1 + d_2)}$$

$$m_1 = \frac{m d_2}{(d_1 + d_2)} \quad \text{--- (4)}$$

$$\text{--- (3)}$$